SPRING 2007 CAS COURSE 3 SOLUTIONS

1. The number of fires in a 3-day period is Poisson with a mean of $\frac{15}{10} = 1.5$. The number of fires in the 3-day period following the 8-th fire is Poisson with a mean of 1.5, so the probability of no fires in that 3-day period is $e^{-1.5} = .223$. Answer: B

2. The total revenue generated in one hour S, has a compound Poisson distribution with Poisson mean of $\lambda = 3 \times 60 = 180$ customers per hour. The severity (revenue per customer) X is Normal with mean $\mu = 50$ and coefficient of variation $\frac{\sigma}{\mu} = .25$, so that $\sigma = 12.5$ and $\sigma^2 = 156.25$. The variance of the total revenue per hour S for this compound Poisson distribution is $Var(S) = \lambda E(X^2)$.
But $E(X^2) = Var(X) + [E(X)]^2 = \sigma^2 + \mu^2 = 156.25 + 50^2 = 2656.25$.
Then, $Var(S) = (180)(2656.25) = 478,125$, and the standard deviation of S is $\sqrt{Var(S)} = 691.5$. Answer: E

3. According to put-call parity, the price of the put should be
Call price + PV of Strike price − Prepaid forward stock price
$= 2.00 + 50e^{-.5(.03)} - 49.70e^{-.5(.02)} = 2.05$.
Since the put is priced at 2.35, we should be able to sell the put at $2.35
and by a synthetic put for $2.05. The arbitrage profit will be $.30.
The synthetic put can be bought by shorting $e^{-.5(.02)}$ shares of stock and receiving
$49.21, and investing $50e^{-.5(.03)} = 49.26$ (lending) at the risk free rate for 6 months
and buying the call for 2.00, for a net cost of $49.26 + 2.00 - 49.21 = 2.05$.
Answer: B

4. According to put-call parity, the price of the put should be
Call price + PV of Strike price − Prepaid forward stock price .
The prepaid forward stock price is the current stock price minus present value of dividends.
This is $29 - .5[[e^{-\frac{2}{12}(.1)} + e^{-\frac{5}{12}(.1)}] = 28.04$.
The put price is $2 + 30e^{-.5(.1)} - 28.04 = 2.50$. Answer: E

5. ${}_{2|3}q_{60} = \frac{\ell_{62}-\ell_{65}}{\ell_{60}} = \frac{7{,}954{,}179-7{,}533{,}964}{8{,}188{,}074} = .05132$. Answer: C

6. $\overset{\circ}{e}_{30} = \int_0^\infty {}_tp_{30}\,dt = \int_0^{70} \frac{s(30+t)}{s(40)}\,dt = \int_0^{70} \frac{1-\frac{(30+t)^2}{10{,}000}}{1-\frac{30^2}{10{,}000}}\,dt$

$= \int_0^{70} \frac{1}{1-\frac{30^2}{10{,}000}}\,dt - \int_0^{70} \frac{\frac{(30+t)^2}{10{,}000}}{1-\frac{30^2}{10{,}000}}\,dt = \frac{70}{1-\frac{30^2}{10{,}000}} - \frac{1}{1-\frac{30^2}{10{,}000}} \cdot \int_0^{70} \frac{(30+t)^2}{10{,}000}\,dt$

$= \frac{70}{.91} - \frac{1}{.91} \cdot \frac{(30+t)^2}{10{,}000}\Big|_{t=0}^{t=70} = \frac{70}{.91} - \frac{1}{.91} \cdot \frac{100}{3} = 40.3$. Answer: B

7. The actuarial present value is

$\ddot{a}_{\overline{5}|} + (\ddot{a}_{xy} - \ddot{a}_{xy:\overline{5}|}) + \frac{1}{2}[(\ddot{a}_x - \ddot{a}_{x:\overline{5}|}) - (\ddot{a}_{xy} - \ddot{a}_{xy:\overline{5}|})] + \frac{1}{2}[(\ddot{a}_x - \ddot{a}_{x:\overline{5}|}) - (\ddot{a}_{xy} - \ddot{a}_{xy:\overline{5}|})]$

$= 4.465 + (12.478 - 4.387)) + \frac{1}{2}[(14.817 - 4.440) - (12.478 - 4.387)]$

$+ \frac{1}{2}[(13.267 - 4.411) - (12.478 - 4.387)] = 14.1$. Answer: D

8. The loss at issue is $L(\pi) = 1000Z - \pi Y = [1000 + \frac{\pi}{d}]Z - \frac{\pi}{d}$.

This follows from the relationship $Y = \frac{1-Z}{d}$ for the discrete whole life annuity random variable Y and the discrete whole insurance random variable Z.

The variance of $L(\pi)$ is

$Var[L(\pi)] = [1000 + \frac{\pi}{d}]^2 \cdot Var(Z) = [1000 + \frac{\pi}{d}]^2 \cdot [{}^2A_{55} - (A_{55})^2]$

$= [1000 + \frac{\pi}{.056604}]^2 \cdot [.13067 - (.30514)^2]$.

These values are from the Illustrative Table.

The standard deviation is $[1000 + \frac{\pi}{.056604}] \cdot \sqrt{[.13067 - (.30514)^2]}$.

In order for this to be less than 250, we must have

$[1000 + \frac{\pi}{.056604}] \cdot \sqrt{[.13067 - (.30514)^2]} < 250$, so that $\pi < 16.4$. Answer: A

9. The present value random variable for the total amount paid is
$S = 500Z_1 + 500Z_2 + \cdots + 500Z_{100}$, where each Z_i is the present value random variable for an insurance of 1 paid at the end of the year of death of a 30-year-old. The Z's are independent of one another. We wish to find F so that $P[S \leq F] = .85$, where S has an (approximate) normal distribution. The mean and variance of S are
$E(S) = 100 \times 500E(Z) = 100 \times 500A_{30} = 5124$, and
$Var(S) = 100 \times 500Var(Z) = 100 \times 500^2 \cdot [^2A_{30} - (A_{30})^2] = 370,196 = (608.4)^2$.
Then, $P[S \leq F] = P[\frac{S-5124}{608.4} \leq \frac{F-5124}{608.4}] = .85$.
$\frac{F-5124}{608.4}$ is the 85-th percentile of the standard normal distribution, which is $\frac{F-5124}{608.4} = 1.04$ (from the standard normal table). Then $F = 5757.$ Answer: C

10. Since $\theta < Y_i$ for each i, it must be the case that $\theta <$ Minimum$[Y_1, Y_2, Y_3, Y_4, ..., Y_n]$
Any of the other answers result in a likelihood that may be 0. Answer: D

11. The log of the pdf is $ln\, f(x) = ln(\theta + 1) + \theta ln\, x$, and the derivative with respect to θ is $\frac{d}{d\theta}\, ln\, f(x) = \frac{1}{\theta+1} + ln\, x$. The derivative with respect to θ of the loglikelihood function is the sum over the x_i's of $\frac{d}{d\theta}\, ln\, f(x_i)$. This is $\frac{d}{d\theta}\, lnL = \Sigma[\frac{1}{\theta+1} + ln\, x_i]$.
Setting this equal to 0 and solving for θ results in the mle of θ,
$\widehat{\theta} = -\frac{n}{\Sigma\, ln\, x_i} - 1 = -\frac{5}{ln.92+ln.79+ln.90+ln.65+ln.86} - 1 = 3.97$. Answer: E

12. I. True. American options have premiums that are greater than or equal to European options.
II. False. As the stock price increases, it is less valuable to exercise the put.
III. False. For a call, the strike price is the price at which the stock can be purchased if the option is exercised. A higher strike price makes the option less valuable.
Answer: A

13. A synthetic T-Bill can be created using a call, a put and the stock.
Long T-Bill with present value $80e^{-.25r} =$ Short Call (80) + Long Put (80) + Stock
$= -6.70 + 1.60 + 85.00 = 79.90$.
Therefore, $e^{-.25r} = .99875$, and $r = .005$. Answer: A

14. The risk neutral probabilities are $p^* = \frac{e^{.05}-.9}{1.1-.9} = .7564$ for an up-step,
and $.2436$ for a down-step. The option values at time 2 are 26 if the stock price is 121, 4 if the stock price is 99, and 0 if the stock price is 81. The expected present value at time 0 using risk neutral probabilities is
$e^{-2(.05)}[26 \cdot (p^*)^2 + 4 \cdot 2 \cdot p^*(1-p^*)] = 14.79$. Answer: D

15. Using the binomial option pricing model in Chapter 10 of the text, we have
$u = e^{(r-\delta)h+\sigma\sqrt{h}} = e^{(.05-.035)(\frac{1}{3})+.3\sqrt{\frac{1}{3}}} = 1.1981$ and
$d = e^{(r-\delta)h-\sigma\sqrt{h}} = e^{(.05-.035)(\frac{1}{3})-.3\sqrt{\frac{1}{3}}} = .8452$.
The risk-neutral probability of an increase in the stock price in the binomial tree is
$p^* = \frac{e^{(r-\delta)h}-d}{u-d} = \frac{e^{(.05-.035)(\frac{1}{3})}-.8452}{1.1981-.8452} = .453$. Answer: C

16. The number of shares of stock in the replicating portfolio is
$\Delta = e^{-\delta h} \cdot \frac{C_u - C_d}{S(u-d)}$, where C_u and C_d are the values of the option if the stock goes up or down.
$C_u = 6$, $C_d = 0$. Since the dividend rate is $\delta = 0$, we have $\Delta = \frac{6-0}{10(1.8-.4)} = .429$.
Answer: C

17. We assume no dividends. Based on the given parameter values
$r = .05$, $\delta = 0$, $\sigma = .35$, $h = 1$, we have $u = e^{(r-\delta)h+\sigma\sqrt{h}} = e^{.05+.35} = 1.4918$
and $d = e^{(r-\delta)h-\sigma\sqrt{h}} = e^{.05-.35} = .7408$.
The risk-neutral probability of an increase in stock price is $p^* = \frac{e^r - d}{u-d} = \frac{e^{.05}-.7408}{1.4918-.7408} = .4134$.
At time 2, the nodes in the binomial tree are
$35u^2 = 35(1.4918)^2 = 77.89$, with risk-neutral probability $(p^*)^2 = (.4134)^2 = .171$, and
$35ud = 35(1.4918)(.7408) = 38.68$, with risk-neutral probability $2(p^*)(1-p^*) = .485$, and
$35d^2 = 35(.7408)^2 = 19.21$ with risk-neutral probability $(1-p^*)^2 = .344$.
The option values are 0 at the nodes $35u^2$and $35ud$, and 12.79 at the node $35d^2$.
The expected present value of the option price at time 0 is
$e^{-2(.05)}[12.79(.344)] = 3.98$. Answer: D

18. We use a chi-square goodness-of-fit test statistic. The null hypothesis is that the sick days are uniformly distributed, with an expected number of 24 for each day of the week (one-fifth of 120). The chi-square statistic is

$\frac{(32-24)^2}{24}+\frac{(18-24)^2}{24}+\frac{(18-24)^2}{24}+\frac{(20-24)^2}{24}+\frac{(32-24)^2}{24}=9.00$.

The statistic has $5-1=4$ degrees of freedom (there was no estimation).

From the chi-square table, we see that 5% of probability is to the right of 9.49 (this is the 95-th percentile) for 4 degrees of freedom. The p-value of the test statistic 9.00 is the probability to the right of 9.00, which is greater than .05. Answer: E

19. The table would be a 4×4 grid with 16 entries.

	Class 1	Class 2	Class 3	Class 4	Total
1 Vehicle	x_{11}	x_{12}	x_{13}	x_{14}	$V1$
2 Vehicles	x_{21}	x_{22}	x_{23}	x_{24}	$V2$
3 Vehicles	x_{31}	x_{32}	x_{33}	x_{34}	$V3$
4 Vehicles	x_{41}	x_{42}	x_{43}	x_{44}	$V4$
	$C1$	$C2$	$C3$	$C4$	T

Each x_{ij} is the number of policyholders with i vehicles in Class j.

In a data set, we would know the totals $V1, V2, V3, V4, C1, C2, C3, C4,$ and T.

Therefore, knowing any 3 entries in a row, and the row total, means we would know the 4th entry in the row, and the same goes for a column. We lose one degree of freedom for each row, and for each column, but because of duplication with a row and column, we lose $4+4-1=7$ degrees of freedom. From the total number of 16 cells, we lose 7 degrees of freedom, for a net degrees of freedom of 9. Answer: A

20. $C=S_0e^{-\delta T}\Phi(d_1)-Ke^{-rT}\Phi(d_2)$.

$S_0=58.96$, $\delta=.05$, $K=60$, $\sigma=.2$, $r=.06$, $T=.25$,

$d_1=\frac{lnS-lnK+(r-\delta+\frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}=\frac{ln58.96-ln60+(.06-.05+\frac{1}{2}(.2)^2)(.25)}{(.2)\sqrt{.25}}=-.10$,

$d_2=d_1-\sigma\sqrt{T}=-.10-(.2)\sqrt{.25}=.20$.

$C=58.96e^{-.05(.25)}\Phi(-.1)-60e^{-.05(.25)}\Phi(-.2)$

$=58.96e^{-.05(.25)}(.4602)-60e^{-.06(.25)}(.4207)=1/93$. Answer: B

21. $P = Ke^{-rT}\Phi(-d_2) - S_0e^{-\delta T}\Phi(-d_1)$.
$K = 8.75$, $r = .08$, $S_0 = 9.67$, $\delta = 0$, $\sigma = .40$, $T = .25$,
$d_1 = \frac{ln9.67 - ln8.75 + (.08 + \frac{1}{2}(.4)^2)(.25)}{(.4)\sqrt{.25}} = .7$, $d_2 = .5$.
$P = 8.75e^{-.(08)(.25)}\Phi(-.5) - 9.67\Phi(-.7)$
$= 8.75(.9802)(.3085) - 9.67(.242) = .31$. Answer: A

22. The significance level is the probability of rejecting the null hypothesis given that it is true. The Type II error is the probability of not rejecting the null hypothesis given that it is not true (given that the alternative is true). The null hypothesis will not be rejected of $\widehat{p}$ is below the critical value, say c. If the alternative hypothesis is true, then the probability of not rejecting the null hypothesis is $P[\widehat{p} < c] = P[z < \frac{c-.75}{\sqrt{(.75)(.25)/625}}] = P[z < \frac{c-.75}{.01732}]$. We are told that this is 11.07%. Therefore, $\frac{c-.75}{.01732}$ is the 11.07-percentile of the standard normal distribution, which is -1.223. Therefore, $\frac{c-.75}{.01732} = -1.223$, so that $c = .7288$.
The level of significance is $P[\widehat{p} > .7288 | p = .7] = P[z > \frac{.7288-.7}{\sqrt{(.7)(.3)/625}}] = P[z > 1.58] = .06$
This would be answer E, which is not in agreement with the answer of B given on the exam.

23. The power of the test is the probability that H_0 is rejected given that H_1 is true.
The significance level is $P[\frac{\bar{X}-70}{\sqrt{100/n}} > c] = .023$, which implies that $c = 2.0$.
The power of the test is $P[\frac{\bar{X}-70}{\sqrt{100/n}} > 2.0 | \mu = 75] = .5$.
This probability can be written as $P[\frac{\bar{X}-75}{\sqrt{100/n}} > 2.0 - \frac{5}{\sqrt{100/n}} | \mu = 75] = .5$,
which implies that $2.0 - \frac{5}{\sqrt{100/n}} = 0$, so that $n = 16$.
To raise the power to 90%, we need $P[\frac{\bar{X}-70}{\sqrt{100/n}} > 2.0 | \mu = 75] = .9$
which can be written as $P[\frac{\bar{X}-75}{\sqrt{100/n}} > 2.0 - \frac{5}{\sqrt{100/n}} | \mu = 75] = .9$.
This implies that $2.0 - \frac{5}{\sqrt{100/n}} = -1.28$, so that $n \geq 43.03$, so we need 44 sample values.
This would require $44 - 16 = 28$ more sample values. Answer: D

24. This probability is ${}_{1/3}q_{50.25} = \frac{(1/3)q_{50}}{1-(.25)q_{50}} = .00196$. Answer: B

25. $T(xy)$ is the time until failure of the joint life status. The joint status has survival probability ${}_tp_{xy} = {}_tp_x \cdot {}_tp_y = (1 - .01t)^2$. Then
$E[T(xy)] = \overset{\circ}{e}_{xy} = \int_0^{100} {}_tp_{xy}\, dt = \int_0^{100} (1 - .01t)^2\, dt = \frac{(1-.01t)^3}{-.03}\Big|_{t=0}^{t=100} = 33.33$.
The second moment of $T(xy)$ is
$E[T(xy)] = \int_0^{100} 2t \cdot {}_tp_{xy}\, dt = \int_0^{100} 2t(1 - .01t)^2\, dt$
$= \int_0^{100} 2t(1 - .02t + .0001t^2)\, dt = 2\int_0^{100} (t - .02t^2 + .0001t^3)\, dt = 1666.67$.
The variance of $T(xy)$ is $1666.67 - (33.33)^2 = 556$.   Answer: A

26. $Cov[T(xy), T(\overline{xy})] = E[T(xy) \cdot T(\overline{xy})] - E\{T(xy)] \cdot E[T(\overline{xy})]$.
We know that $E[T(x)] = \frac{1}{\mu}$ and $E[T(y)] = \frac{1}{\alpha}$, and $E\{T(xy)] = \frac{1}{\alpha+\mu}$, since ${}_tp_{xy} = {}_tp_x \cdot {}_tp_y = e^{-\mu t} \cdot e^{-\alpha t} = e^{-(\alpha+\mu)t}$.
$E[T(xy) \cdot T(\overline{xy})] = E\{T(x)T(y)] = E[T(x)] \cdot E[T(y)] = \frac{1}{\mu} \cdot \frac{1}{\alpha}$
(this is true since one of $T(xy)$ and $T(\overline{xy})$ is $T(x)$ and the other is $T(y)$).
Also, $E[T(\overline{xy})] = E[T(x)] + E[T(y)] - E[T(xy)] = \frac{1}{\mu} + \frac{1}{\alpha} - \frac{1}{\alpha+\mu}$.
Then,
$Cov[T(xy), T(\overline{xy})] = E[T(xy) \cdot T(\overline{xy})] - E\{T(xy)] \cdot E[T(\overline{xy})]$
$= \frac{1}{\mu} \cdot \frac{1}{\alpha} - (\frac{1}{\alpha+\mu})(\frac{1}{\mu} + \frac{1}{\alpha} - \frac{1}{\alpha+\mu}) = \frac{1}{\alpha\mu} - \frac{(\alpha+\mu)^2 - \alpha\mu}{\alpha\mu(\alpha+\mu)^2} = \frac{1}{(\alpha+\mu)^2}$.   Answer: C

27. ${}_4d_{65}^{(2)} = d_{65}^{(2)} + d_{66}^{(2)} + d_{67}^{(2)} + d_{68}^{(2)}$.
$d_{65}^{(2)} = \ell_{65}^{(\tau)} \cdot q_{65}^{(2)} = 1000(.2) = 200$.
$d_{66}^{(2)} = \ell_{66}^{(\tau)} \cdot q_{66}^{(2)} = \ell_{65}^{(\tau)} \cdot p_{65}^{(\tau)} \cdot q_{66}^{(2)} = 1000(.75)(.3) = 225$.
$d_{67}^{(2)} = \ell_{67}^{(\tau)} \cdot q_{67}^{(2)} = \ell_{65}^{(\tau)} \cdot {}_2p_{65}^{(\tau)} \cdot q_{67}^{(2)} = 1000(.75)(.6)(.4) = 180$.
$d_{68}^{(2)} = \ell_{68}^{(\tau)} \cdot q_{68}^{(2)} = \ell_{65}^{(\tau)} \cdot {}_3p_{65}^{(\tau)} \cdot q_{68}^{(2)} = 1000(.75)(.6)(.45)(.5) = 101.25$.
${}_4d_{65}^{(2)} = 200 + 225 + 180 + 101.25 = 706.25$.   Answer: B

28. The subject must transition from State 2 at time 0 to State 3 at time 2 and then transition to State 1 at time 3. This is ${}_2Q_0^{(2,3)} \cdot Q_2^{(3,1)}$. The matrix Q_n is the transition matrix from time $n-1$ to time n (which is not the same as the notation used in the study note; the study note would denote this matrix Q_{n-1}). $Q_2^{(3,1)} = .25$ is found from the matrix Q_3 .
${}_2Q_0^{(2,3)}$ is the 2-3 entry in the matrix product $Q_1 \times Q_2$. This will be
${}_2Q_0^{(2,3)} = (.35)(.25) + (.50)(.20) + (.15)(.40) = .2475$.
The probability in question is ${}_2Q_0^{(2,3)} \cdot Q_2^{(3,1)} = (.2475)(.25) = .061875$.   Answer: B

29. The t-statistic for testing for a one-sided test of the equality of the means of two normal distributions with equal but unknown variances is $t = \dfrac{\bar{y}-\bar{x}}{s_p \cdot \sqrt{\frac{1}{n_X} + \frac{1}{n_Y}}}$,

where $s_p^2 = \dfrac{(n_X-1)s_X^2 + (n_Y-1)s_Y^2}{n_X + n_Y - 2}$.

We are given $\overline{x} = 10$, $s_X^2 = 4$, $n_X = 11$, $\overline{y} = 9$, $s_Y^2 = 12$, $n_Y = 11$.

$s_p^2 = \frac{(11-1)(4)+(11-1)(12)}{11+11-2} = 8$ and $t = \frac{9-10}{\sqrt{8}\cdot\sqrt{\frac{1}{11}+\frac{1}{11}}} = -.829$.

The t-statistic has $n_X + n_Y - 2 = 20$ degrees of freedom in the t-distribution. The critical value for a one-sided test with significance level .05 is $T = 1.725$ (found from the t-table with 0.1 area in both tails, so as to have .05 area in one tail). The absolute difference is $|t - T| = |-.829 - 1.725| = 2.554$.

This is answer E, which is not the answer of C indicated on the exam.

30. The test statistic for the comparison of the variances of two normal distributions is $f = \frac{s_1^2}{s_2^2} = \frac{16}{9}$. The critical value for a one-sided test is found in the F-distribution table with $n_X - 1 = 7$ and $n_Y - 1 = 8$ degrees of freedom. With significance level .05, the critical value is $F = 3.50$. The absolute difference is $|f - F| = |\frac{16}{9} - 3.50| = 1.72$. This is answer C. The exam indicated that the answer is A.

31. $R^2 = 1 - \frac{ESS}{TSS}$, where $TSS =$ total sum of squares $= \Sigma(y_i - \overline{y})^2 = \Sigma y_i^2 - n\overline{y}^2$.

For this data set, $\overline{y} = 13$ and $n = 5$, so $TSS = 34$.

$ESS =$ error sum of squares $= \Sigma(y_i - \widehat{y}_i)^2$

$= (10 - 9.6)^2 + \cdots + (17 - 16.4)^2 = 5.1$.

$R^2 = 1 - \frac{5.1}{34} = .85$. Answer: D

32. In order to delta-hedge his position, the market maker buys $10,000\Delta$ shares.

From the given information, $\Delta = e^{-\delta T}N(d_1) = e^{-.07}(.5793) = .5401$.

The market maker buys $5,401$ shares of stock.

The market maker loses $100 \times 56.08 = 5608$ as a result of the option price change ("one option" refers to a contract of an option on 100 shares). The market maker gains 5401 as a result of the $1 increase in his 5401 shares. The net profit is

$5401 - 5608 = -207$. Answer: C

33. I. False. The value of a written call is proportional in gamma to the square of the price change. If the gamma of a call is positive, then a written call will lose money.
II. True. Negative theta indicates that a call becomes less valuable as time progresses. The call option writer benefits as the value of the call decreases.
III. False. A market maker hedges a put by selling stock.
II only is not an answer choice. The answer indicated on the exam is D.

34. The combination of a down-and-in call option and a down-and-out call option, each with strike price K, is an ordinary call with strike price K. This is true because the combination eliminates the uncertainty regarding whether or not the stock crosses the down barrier. Therefore, the price of an ordinary call with strike price K is $25+30=55$.
The same logic applies to the combination of and up-and-out call and an up-and-in call with the same strike price K. They combine to be an ordinary call with strike price K, so the price of an ordinary call with strike price K is $15+X$. Since all strike prices of the barrier options are the same, it must be true that $15+X=55$, so that $X=40$. Answer: E

35. There appears to be a typographical error in the expression for $S(t)$. The $\sigma Z(t)$ factor should also be in the exponent. If so, the following solution applies. As it stands, the question has an error in the statement.
According to Ito's Lemma, if C is a function of S and t, and $S(t)$ follows geometric Brownian motion of the form given in the first bullet of the problem, then
$dC = [(\alpha-\delta)\,S(t)\,\frac{d}{dS}C \,+\, \frac{1}{2}\sigma^2\,\frac{d^2}{dS^2}\,C \,+\, \frac{d}{dt}\,C]\,dt \,+\, \sigma S\,\frac{d}{dS}C\,dZ$.
Since $C=S$, it follows that $\frac{d}{dS}C=1$, and $\frac{d^2}{dS^2}S=0$ and $\frac{d}{dt}C=0$.
Then, $dC = [(\alpha-\delta)\,S]\,dt \,+\, \sigma S\,dZ$.
This is answer B, when we cancel $-\frac{1}{2}\sigma^2 S(t)\,dt$ with $\frac{1}{2}\sigma^2 S(t)\,dt$. Answer: B

36. This problem has a couple of errors (besides the misspelling of Vasicek).
The Vasicek model has parameters a, b, r, σ. Perhaps the "r" in the text looks like the Greek letter γ, and that may account for the problem using γ instead of r. This question is fashioned after the calculations in Table 24.1 on page 786 of the McDonald textbook. The intention of this question was to find $a(b-r)=.15(.10-.05)=.0075$, which is the approximate expected rate of change in the interest rate per unit time. The question asks for the "expected change in the interest rate", which would be $a(b-r)\,dt$ (since the $E[dz]$ factor would be 0 for the Gaussian process z). The exam gives the answer as E, consistent with the numerical value of .0075, but the problem seems to be defective.

37. According to the Black-Derman-Toy model, the yield volatility at time h at a particular node in the binomial tree is $0.5 \times ln[\frac{y(h,T,r_u)}{y(h,T,r_d)}]$, where $y(h,T,r_u)$ and $y(h,T,r_d)$ are the yields to maturity at the upper and lower nodes of the branch. The bond originally matured in 3 years, so at time 1, there are still 2 years remaining until maturity. The upper branch at time 1 has bond price .8133, which has an annualized yield for the two remaining years that is $(\frac{1}{.8133})^{1/2} - 1 = .1089$.

The lower branch has annualized yield $(\frac{1}{.8537})^{1/2} - 1 = .0823$.

The yield volatility is $.5 \cdot ln(\frac{.1089}{.0823}) = .140$. Answer: B

38. The retrospective form of the reserve is ${}_5V_{40:\overline{20}|} = P_{40:\overline{20}|} \cdot \ddot{s}_{40:\overline{5}|} - \frac{A^{\ 1}_{40:\overline{5}|}}{{}_5E_{40}}$.

We are given $A^{\ 1}_{40:\overline{5}|} = 0.060$ and ${}_5E_{40} = 0.694$, and $\ddot{s}_{40:\overline{5}|} = \frac{\ddot{a}_{40:\overline{5}|}}{{}_5E_{40}} = 6.261$.

$P_{40:\overline{20}|} = \frac{A_{40:\overline{20}|}}{\ddot{a}_{40:\overline{20}|}} = \frac{A^{\ 1}_{40:\overline{20}|} + {}_{20}E_{40}}{\ddot{a}_{40:\overline{20}|}} = .03571$.

Then, ${}_5V_{40:\overline{20}|} = (.03571)(6.261) - \frac{.060}{.694} = .1371$. Answer: C

39. The second bullet point indicates that the survival model follows DeMoivre's Law with $\omega = 100$. Under this law, continuous whole life insurance values have the form $\overline{A}_x = \frac{1}{\omega - x} \cdot \overline{a}_{\overline{\omega - x}|}$, where $\overline{a}_{\overline{\omega - x}|} = \frac{1 - v^{\omega - x}}{\delta}$, and $\delta = ln(1+i)$ is the force of interest.

We use the reserve form ${}_t\overline{V}(\overline{A}_x) = \frac{\overline{A}_{x+t} - \overline{A}_x}{1 - \overline{A}_x}$.

Then, ${}_{20}\overline{V}(\overline{A}_{35}) = \frac{\overline{A}_{55} - \overline{A}_{35}}{1 - \overline{A}_{35}}$.

From DeMoivre's Law, we have $\overline{A}_{35} = \frac{1}{100-35} \cdot \overline{a}_{\overline{100-35}|} = \frac{1}{65} \cdot \frac{1 - v^{65}}{ln(1.05)} = .3021$,

and $\overline{A}_{55} = \frac{1}{45} \cdot \frac{1 - v^{45}}{ln(1.05)} = .4048$.

Then, ${}_{20}\overline{V}(\overline{A}_{35}) = \frac{.4048 - .3021}{1 - .3021} = .147$. Answer: A

40. The driver starts out as Preferred. The only possible cost at the end of the first year is 20 and this occurs if the driver transfers from Preferred to Standard. That probability is .3, found in Q_1. The actuarial present value of the first year end transition cost is $20v(.3) = 5.71$.

The possible transition costs at the end of the second year are 20 and 10.
20 occurs if the transition is from P to S at the end of the 2nd year, which has probability $(.7)(.2) = .14$ (path P-P-S).
10 occurs if the transition is from S to P at the end of the 2nd year, which has probability $(.3)(.2) = .06$ (path P-S-P).
The APV of 2nd year end transition costs is $[(20)(.14) + (10)(.06)]v^2 = 3.08$.

The possible transition costs at the end of the third year are 20 and 10.
20 occurs if the transition is from P to S at the end of the 3rd year, which has probability ${}_2Q_0^{(P.P)} \cdot (.1) = [(.7)(.8) + (.3)(.2)](.1) = .062$ (paths P-P-P-S and P-S-P-S).
10 occurs if the transition is from S to P at the end of the 3rd year, which has probability ${}_2Q_0^{(P.S)} \cdot (.2) = [(.7)(.2) + (.3)(.8)](.2) = .076$ (paths P-P-S-P and P-S-S-P) .
The APV of 3rd year end transition costs is $[(20)(.062) + (10)(.076)]v^3 = 1.73$.

The total APV is $5.71 + 3.08 + 1.73 = 10.52$. Answer: D