EXAM M QUESTIONS OF THE WEEK

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Week of April 24/06

A homogeneous Markov chain $\{X_n : n \geq 0\}$ has states 0,1,2. The one-step transition probability matrix is $\boldsymbol{Q} = \begin{bmatrix} 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 \\ 0 & 1 & 0 \end{bmatrix}$.

Find the unconditional distribution of X_1 assuming that $P(X_0 = 0) = P(X_0 = 1) = \frac{1}{2}$ and $P(X_0 = 2) = 0$.

The solution can be found below.

Week of April 24/06 - Solution

$$P(X_1 = 0) = P(X_1 = 0|X_0 = 0) \cdot P(X_0 = 0)$$
$$+ P(X_1 = 0|X_0 = 1) \cdot P(X_0 = 1) + P(X_1 = 0|X_0 = 2) \cdot P(X_0 = 2)$$
$$= Q^{(0,0)} \cdot P(X_0 = 0) + Q^{(1,0)} \cdot P(X_0 = 1) + Q^{(2,0)} \cdot P(X_0 = 2)$$
$$= (0)(\tfrac{1}{2}) + (\tfrac{2}{3})(\tfrac{1}{2}) + (0)(0) = \tfrac{1}{3}\ .$$

$$P(X_1 = 1) = P(X_1 = 1|X_0 = 0) \cdot P(X_0 = 0)$$
$$+ P(X_1 = 1|X_0 = 1) \cdot P(X_0 = 1) + P(X_1 = 1|X_0 = 2) \cdot P(X_0 = 2)$$
$$= Q^{(0,1)} \cdot P(X_0 = 0) + Q^{(1,1)} \cdot P(X_0 = 1) + Q^{(2,1)} \cdot P(X_0 = 2)$$
$$= (\tfrac{1}{3})(\tfrac{1}{2}) + (\tfrac{1}{3})(\tfrac{1}{2}) + (1)(0) = \tfrac{1}{3}\ .$$

$$P(X_1 = 2) = P(X_1 = 2|X_0 = 0) \cdot P(X_0 = 0)$$
$$+ P(X_1 = 2|X_0 = 1) \cdot P(X_0 = 1) + P(X_1 = 2|X_0 = 2) \cdot P(X_0 = 2)$$
$$= Q^{(0,2)} \cdot P(X_0 = 0) + Q^{(1,2)} \cdot P(X_0 = 1) + Q^{(2,2)} \cdot P(X_0 = 2)$$
$= (\tfrac{2}{3})(\tfrac{1}{2}) + (0)(\tfrac{1}{2}) + (0)(0) = \tfrac{1}{3}$ (alternatively, $P(X_1 = 2) = 1 - P(X_1 = 0 \text{ or } 1)$) .