EXAM C QUESTIONS OF THE WEEK

S. Broverman, 2006

Week of April 3/06

The parameter λ has a prior distribution with pdf $\pi(\lambda) = 2\lambda$ for $0 < \lambda < 1$.
The conditional distribution of X given λ is uniform on the interval $(0, \lambda)$.
Find the posterior density $\pi(\lambda|x)$.

Solution can be found below.

Week of April 3/06 - Solution

The model density is $f(x|\lambda) = \frac{1}{\lambda}$ for $0 < x < \lambda$.

The joint density is $f(x, \lambda) = f(x|\lambda) \cdot \pi(\lambda) = \frac{1}{\lambda} \cdot 2\lambda = 2$

on the triangular region $0 < x < \lambda < 1$.

The marginal density of X is

$f_X(x) = \int_x^1 f(x, \lambda) \, d\lambda = \int_x^1 2 \, d\lambda = 2(1 - x)$ for $0 < x < 1$.

The posterior density if

$\pi(\lambda|x) = \frac{f(x,\lambda)}{f_X(x)} = \frac{2}{2(1-x)} = \frac{1}{1-x}$ for $x < \lambda < 1$ (this is uniform on the interval $(x, 1)$).