EXAM FM QUESTIONS OF THE WEEK

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Week of August 14/06

A loan of $$L$ is to be repaid over a 10 year period. The borrower is offered two different repayment plans from which to choose.

Repayment Plan A: Interest only is paid to the lender at the end of each year at annual effective rate 10%. Principal is accumulated in a sinking fund with level annual deposits at the end of each year for 10 years, and the full principal of L is paid at the end of 10 years. The sinking fund earns interest at annual effective rate 5%.

Repayment Plan B: Interest only is paid to the lender at the end of each year at annual effective rate 10%. Principal is accumulated in a sinking fund with level annual deposits at the end of each year for 10 years. At the end of 5 years, $L/2$ of principal is paid to the lender, and at the end of 10 years, the other $L/2$ of principal is paid to the lender.

The total interest paid by the borrower in the two plans is I_A and I_B, respectively. Find I_A/I_B .

The solution can be found below.

Week of August 14/06 - Solution

Plan A: The sinking fund deposit is $\frac{L}{s_{\overline{10}|.05}} = .079505L$.

Total paid by borrower over 10 years is $10L(.1) + 10(.079505L) = 1.79505L$.

Total interest paid by borrower under Plan A over 10 years is $.79505L = I_A$.

Plan B: The sinking fund deposit is $\frac{L/2}{s_{\overline{5}|.05}} = .090487L$.

The reason for this is that this accumulates to the $L/2$ of principal paid at the end of 5 years, and then accumulates to the $L/2$ of principal paid at the end of 10 years.

Total paid by borrower over 10 years is $5L(.1) + 5(\frac{L}{2})(.1) + 10(.090487L) = 1.65487L$.

The reason for this is that the interest payments made to the lender drop to $(\frac{L}{2})(.1)$ per year after the first 5 years since the amount owing has dropped to $L/2$.

Total interest paid by borrower under Plan B over 10 years is $.65487L = I_B$.

$I_A/I_B = 1.214$.