EXAM P QUESTIONS OF THE WEEK

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Question 4 - Week of August 15

A random variable X has a probability mass of 0.2 at $X = 0$ and a probability mass of 0.1 at $X = 1$. For all other values, X has the following density function:

$$f_X(x) = \begin{cases} 0 & x < 0 \\ x & 0 < x < 1 \\ 2x & 1 < x < c \\ 0 & x \geq c \end{cases}$$, where c is a constant.

(a) Find $P(.5 < X < 1)$

(b) Find $P(X < 1 | X > .5)$

(c) Find the mean of X.

The solution can be found below.

Question 4 Solution

(a) $P(.5 < X < 1) = \int_{.5}^{1} x\,dx \;\; = \; .375$.

(b) $P(X < 1|X > .5) = \frac{P(.5<X<1)}{P(X>.5)}$.

From part (a), the numerator is $.375$.

Although we have not determined the value of c, we know that

$P(X > .5) = 1 - P(X \leq .5) = 1 - [P(X = 0) + P(0 < X < \; \leq .5)]$

$= 1 - [.2 \, + \, \int_0^{.5} x\,dx] \; = 1 - .325 = .675$.

Then $\;P(X < 1|X > .5) = \frac{P(.5<X<1)}{P(X>.5)} = \frac{.375}{.675} = .556$.

(c) In order for X to be a properly defined random variable, total probability over the probability space must be 1. Total probability over the probability space is found by adding the probabilities at the points of probability mass and integrating the density function over the continuous region for the random variable. In this problem the total probability is

$P[X = 0] \, + \, \int_0^1 x\,dx \, + \, P[X = 1] \, + \, \int_1^c 2x\,dx \; = 0.2 + 0.5 + 0.1 + (c^2 - 1) = c^2 - .2$.

In order for this total probability to be 1, we must have $c^2 - .2 = 1$, or equivalently

$c = (1.2)^{.5} = 1.0954$.

The mean of X is $\Sigma x \cdot P(X = x) \, + \, \int x\,f(x)\,dx$,

where the sum is taken over probability mass points and the integral is over the continuous region.

$E[X] = (0) \cdot P(X = 0) + (1) \cdot P(X = 1) + \int_0^1 x \cdot x\,dx + \int_1^{1.2^{.5}} x \cdot 2x\,dx$

$= 0 + (1)(.1) + \frac{1}{3} + \frac{2[(1.2^{.5})^3 - 1]}{3} = .643$.