EXAM FM QUESTIONS OF THE WEEK

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Question 2 - Week of August 1

You have seen the basic annuity relationships

$$a_{\overline{n}|i} \cdot (1+i)^n = s_{\overline{n}|i} \quad \text{and} \quad s_{\overline{n}|i} \cdot v_i^n = a_{\overline{n}|i}$$

Rank the following in numerical order, from smallest to largest

$$a_{\overline{20}|.05} \cdot (1.10)^{20} \ , \ a_{\overline{20}|.10} \cdot (1.05)^{20} \ , \ s_{\overline{20}|.05} \ , \ s_{\overline{20}|.10} \ .$$

Show by general reasoning that the ordering must always be as you found it if .05 and .10 are replaced by positive interest rates i and j with $i < j$.

Rank the following numerical order, from smallest to largest

$$s_{\overline{20}|.05} \cdot v_{.10}^{20} \ , \ s_{\overline{20}|.10} \cdot v_{.05}^{20} \ , \ a_{\overline{20}|.05} \ , \ a_{\overline{20}|.10} \ .$$

Show by general reasoning that the ordering must always be as you found it if .05 and .10 are replaced by positive interest rates i and j with $i < j$.

The solution can be found below.

Question 2 Solution

$a_{\overline{20}|.05} \cdot (1.10)^{20} = 83.84$, $a_{\overline{20}|.10} \cdot (1.05)^{20} = 22.59$, $s_{\overline{20}|.05} = 33.07$, $s_{\overline{20}|.10} = 57.27$.

We know that $a_{\overline{n}|j} \cdot (1+j)^n = s_{\overline{n}|j}$ it follows that if $i < j$ then $a_{\overline{n}|j} \cdot (1+i)^n < a_{\overline{n}|j} \cdot (1+j)^n = s_{\overline{n}|j}$. This shows that $a_{\overline{20}|.10} \cdot (1.05)^{20} < s_{\overline{20}|.10}$.

We know that if $i < j$ then $(1+i)^k < (1+j)^k$ for $k = 1, 2, \ldots$. It follows that $s_{\overline{n}|i} < s_{\overline{n}|j}$, showing that $s_{\overline{20}|.05} < s_{\overline{20}|.10}$.

We know that if $i < j$ then $v_j^k < v_i^k$, so that $a_{\overline{n}|j} < a_{\overline{n}|i}$. It follows that $s_{\overline{n}|j} = a_{\overline{n}|j} \cdot (1+j)^n < a_{\overline{n}|i} \cdot (1+j)^n$, so that $s_{\overline{20}|.10} < a_{\overline{20}|.05} \cdot (1.10)^{20}$.

$s_{\overline{20}|.05} \cdot v_{.10}^{20} = 4.92$, $s_{\overline{20}|.10} \cdot v_{.05}^{20} = 21.6$, $a_{\overline{20}|.05} = 12.5$, $a_{\overline{20}|.10} = 8.5$.

Similar arguments to those given above apply to these values.