EXAM M QUESTIONS OF THE WEEK

S. Broverman, 2005

Question 2 - Week of August 1

An insurer creates a discrete single premium insurance policy with issue age (35) for which the death benefit will be 200,000 if death occurs before age 65 and 100,000 if death occurs after age 65. Mortality follows the Illustrative Life Table at a 6% rate of interest. The insurer issues this policy to 1000 independent individuals and charges a premium per policy that results in a 90% probability that premiums will be enough to cover total cost of benefits based on a normal approximation to the distribution of the present value of total cost of benefits. Find the premium per policy.

The solution can be found below.

Question 2 Solution

We will denote by W the present value random variable for one policy. Then
$E[W] = 200,000A_{35} - 100,000v^{30}\,{}_{30}p_{35}A_{65}$
(we can also express this as $200,000A_{\overset{1}{35}:\overline{30}|} + 100,000v^{30}\,{}_{30}p_{35}A_{65}$, but the first expression is usually more convenient for calculation).

From the Illustrative Table at 6% we have $A_{35} = .12872$, $A_{65} = .43980$ and
$v^{30}\,{}_{30}p_{35} = v^{20}\,{}_{20}p_{35} \cdot v^{10}\,{}_{10}p_{55} = {}_{20}E_{35} \cdot {}_{10}E_{55} = (.28600)(.48686) = .13924$.
Then $E[W] = 19,620$.

In order to solve the problem, we need to know $Var[W] = E[W^2] - (E[W])^2$.
Since W^2 is found by squaring the benefit and the present value factor, we have
$E[W^2] = 200,000^2 \cdot {}^2A_{\overset{1}{35}:\overline{30}|} + 100,000^2 \cdot v^{60}\,{}_{30}p_{35} \cdot {}^2A_{65}$, which can also be written as
$E[W^2] = 200,000^2 \cdot {}^2A_{35} - (200,000^2 - 100,000^2) \cdot v^{60}\,{}_{30}p_{35} \cdot {}^2A_{65}$
(because ${}^2A_{\overset{1}{35}:\overline{30}|} = {}^2A_{35} - \cdot v^{60}\,{}_{30}p_{35} \cdot {}^2A_{65}$).

From the Illustrative Table, ${}^2A_{35} = .03488$, ${}^2A_{65} = .23603$ and
$v^{60}\,{}_{30}p_{35} = v^{30} \cdot v^{30}\,{}_{30}p_{35} = (.17411)(.13924) = .02424$.

Then,
$E[W^2] = 200,000^2(.03488) - (200,000^2 - 100,000^2)(.02424)(.23603) = 1,223,558,984$.

Then, $Var[W] = 1,223,558,984 - (19,620)^2 = 838,614,584$.

The total present value of benefit costs is $S = W_1 + W_2 + \cdots + W_{1000}$ for the 1000 policies, so $E[S] = 1000E[W] = 1000 \times 19,620 = 19,620,000$ and
$Var[S] = 1000Var[W] = 838,614,584,000$.
We first find C so that $P[S \leq C] = .90$. If C is the total premium for all 1000 policies, then there will be a 90% probability that premium is enough to cover benefit costs.

We can write the probability in the form $P[\frac{S-E[S]}{\sqrt{Var[S]}} \leq \frac{C-E[S]}{\sqrt{Var[S]}}] = .90$.

According to the normal approximation applied to S, $\frac{S-E[S]}{\sqrt{Var[S]}}$ has a standard normal distribution, and therefore $\frac{C-E[S]}{\sqrt{Var[S]}}$ is the 90th percentile of the standard normal, which is 1.282 (from interpolation in the normal table from the Exam M tables).

Therefore $\frac{C-E[S]}{\sqrt{Var[S]}} = \frac{C-19,620,000}{\sqrt{838,614,584,000}} = 1.282$ from which we get $C = 20,794,003$.

This is the premium for 1000 policies, so the premium per policy is $20,794$.