EXAM FM QUESTIONS OF THE WEEK

S. Broverman, 2006

Week of August 21/06

The following information is given regarding two bonds. Both bonds have a face and redemption amount of 100, both have annual coupons and both bonds are priced at effective annual rate of interest j.

Bond 1: coupon rate r, matures in n years, price is 76.60.

Bond 2: coupon rate $2r$, matures in $2n$ years, price is 118.51.

Find the price of a bond based on effective annual yield rate j, face amount 100, coupon rate $3r$, maturing in $3n$ years.

The solution can be found below.

Week of August 21/06 - Solution

For bond 1, we have $76.60 = 100v^n + 100r \cdot a_{\overline{n}|j}$.

For bond 2 we have $118.51 = 100v^{2n} + 200r \cdot a_{\overline{2n}|j}$.

We get the relationship $\frac{118.51 - 100v^{2n}}{76.60 - 100v^n} = \frac{200r \cdot a_{\overline{2n}|j}}{100r \cdot a_{\overline{n}|j}}$.

Since $a_{\overline{2n}|j} = (1 + v^n) \cdot a_{\overline{n}|j}$, the equation above can be written as $\frac{118.51 - 100v^{2n}}{76.60 - 100v^n} = 2(1 + v^n)$.

This becomes a quadratic equation in $y = v^n$: $118.51 - 100y^2 = 2(76.60 - 100y)(1 + y)$, or $100y^2 + 46.80y - 34.69 = 0$.

Solving for y results in $y = .3998$ or $-.8678$. We ignore the negative root, because we want $y = v^n > 0$. From the first bond price equation, we get $76.60 - 39.98 = 36.62 = 100r \cdot a_{\overline{n}|j}$.

Using the fact that $a_{\overline{3n}|j} = (1 + v^n + v^{2n}) \cdot a_{\overline{n}|j}$, we get the third bond price as $100v^{3n} + 300r \cdot a_{\overline{3n}|j} = 100(.3998)^3 + 3(1 + .3998 + .3998^2)(36.62) = 177.73$.