EXAM FM QUESTIONS OF THE WEEK

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Question 5 - Week of August 22

Perpetuity A has the following sequence of annual payments beginning on January 1, 2005:

$1 , 3 , 5 , 7 , \ldots$

Perpetuity B is a level perpetuity of 1 per year, also beginning on January 1, 2005.

Perpetuity C has the following sequence of annual payments beginning on January 1, 2005:

$1 \ , \ 1+r \ , \ (1+r)^2 \ , \ (1+r)^3 \ , \ \ldots$

On January 1, 2005, the present value of Perpetuity A is 25 times as large as the present value of Perpetuity B, and the present value of Perpetuity A is equal to the present value of Perpetuity C. Based on this information, find r.

The solution can be found below.

Question 5 Solution

Perpetuity A is illustrated in the following time line. Time 0 (valuation date) corresponds to 1/1/05 and time is measured in years.

0	1	2	3	4	$\cdots$
1	3	5	7	9	$\cdots$

This series can be represented as the combination of the following two series:

	0	1	2	3	4	$\cdots$
Series 1	1	1	1	1	1	$\cdots$
Series 2		2	4	6	8	$\cdots$

Suppose that the annual effective rate of interest is i. The pv at time 0 of Series 1 is $\ddot{a}_{\overline{\infty}|i} = \frac{1}{d} = \frac{1+i}{i}$ (a level perpetuity-due of 1 per year).

The pv (at time 0) of the Series 2 is $2(Ia)_{\overline{\infty}|i} = 2(\frac{1}{i} + \frac{1}{i^2})$ (2 $\times$ an increasing perpetuity-immediate).

The total pv of Perpetuity A is $\frac{1+i}{i} + 2(\frac{1}{i} + \frac{1}{i^2}) = 1 + \frac{3}{i} + \frac{2}{i^2}$.

The pv of the Perpetuity B is $\ddot{a}_{\overline{\infty}|i} = \frac{1}{d} = \frac{1+i}{i} = 1 + \frac{1}{i}$ (a level perpetuity-due of 1 per year).

We are given that $1 + \frac{3}{i} + \frac{2}{i^2} = 25(1 + \frac{1}{i})$. This equation can be written in the form $24 + \frac{22}{i} - \frac{2}{i^2} = 0$, or equivalently, in the form $12i^2 + 11i - 1 = 0$. Solving by quadratic formula or by factoring $(12i^2 + 11i - 1 = (12 - i)(1 + i)$), we get two roots for i: $i = \frac{1}{12}$ or -1. We discard the negative root for i, so that $i = \frac{1}{12} = .0833$.

The pv of Perpetuity C is $\frac{1+i}{i-r}$ (this follows from the fact that a geometric perpetuity-immediate with payments 1 , $1+r$, $(1+r)^2$, $(1+r)^3$, $\ldots$ has present value $\frac{1}{i-r}$ one period before the first payment, so that the pv is $(1+i) \cdot \frac{1}{i-r}$ at the time of the first payment).

The pv of Perpetuity A is $1 + \frac{3}{i} + \frac{2}{i^2} = 1 + 3(12) + 2(12^2) = 325$.

Therefore, $\frac{1+\frac{1}{12}}{\frac{1}{12}-r} = 325$, from which we get $r = .080$.