EXAM C QUESTIONS OF THE WEEK

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You are given the following random sample of 6 observations:

  8.0 , 1.8 , 6.9 , .5 , 4.1 , 2.2

The data set is drawn from an exponential distribution.

A likelihood ratio test is applied to test the null hypothesis that the mean of the distribution is θ_0.
For which of the following values of θ_0 does the test reject the null hypothesis at the 10% level
of significance?

  θ_0: 2 , 4 , 6 , 8

Solution can be found below.

Week of August 28/06 - Solution

The likelihood ratio test of $H_0 : \theta = \theta_0$ versus $H_1 : \theta \neq \theta_0$
has test statistic $T = 2ln(\frac{L_1}{L_0}) = 2[\,ln\,L_1 - ln\,L_0\,]$,
where $ln\,L_1$ is the loglikelihood based on the maximum likelihood estimate of θ, and $ln\,L_0$
is the loglikelihood based on the null hypothesis value.

For the exponential distribution with one parameter, the test statistic will have a chi-squared distribution
with 1 degree of freedom. The critical value found for a 10% level of significance found from the chi-
square table is $\chi^2_{.1}(1) = 2.706$. The null hypothesis will be rejected at the 10% level if $2[\,ln\,L_1 - ln\,L_0\,] > 2.706$.

The maximum likelihood estimate of the mean of the exponential distribution is 3.92 (it is the
sample mean). Since $f(x) = \frac{1}{\theta}e^{-x/\theta}$ is the pdf, the log of the pdf is
$ln\,f(x) = -ln\,\theta - \frac{x}{\theta}$, and the loglikelihood is
$ln\,L = -n\,ln\,\theta - \frac{\Sigma x_i}{\theta}$.
Using the sample values and the mle estimate of θ, we get
$ln\,L_1 = -6\,ln\,3.92 - \frac{23.5}{3.92} = -14.19$.
The null hypothesis will be rejected if $2[\,ln\,L_1 - ln\,L_0\,] > 2.706$, which is equivalent to
$2[\,-14.19 - ln\,L_0\,] > 2.706$, which is equivalent to $ln\,L_0 < -15.55$.

For a hypothesized value of θ_0 , the loglikelihood is
$ln\,L_0 = -6\,ln\,\theta_0 - \frac{23.5}{\theta_0}$.
The loglikelihood for each of the stated values of θ_0 and the test results are:

θ_0	$ln\,L_0$	Test Result
2	-15.9	Reject H_0
4	-14.2	Don't reject H_0
6	-14.7	Don't reject H_0
8	-15.4	Don't reject H_0