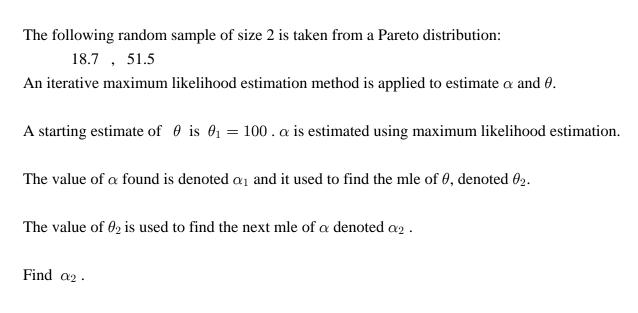
## EXAM C QUESTIONS OF THE WEEK

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## Week of August 7/06



Solution can be found below.

## Week of August 7/06 - Solution

The log of the density of the Pareto distribution is

$$ln f(x) = ln \alpha + \alpha ln \theta - (\alpha + 1) ln(x + \theta)$$
 and

$$\frac{\partial}{\partial \theta} \ln f(x) = \frac{\alpha}{\theta} - \frac{\alpha+1}{x+\theta}$$
 ,  $\frac{\partial}{\partial \alpha} \ln f(x) = \frac{1}{\alpha} + \ln \theta - \ln(x+\theta)$  .

Given  $\theta_1 = 100$ , with 2 sample values, to find the mle of  $\alpha$ , we solve

$$\frac{\partial}{\partial \alpha} \ln L = \frac{2}{\alpha} + 2 \ln \theta - \Sigma \ln(x_i + \theta) = \frac{2}{\alpha} + 2 \ln 100 - (\ln 118.7 + \ln 151.5) = 0.$$

Solving for  $\alpha$  results in  $\alpha_1 = 3.41$ .

Now, with  $\alpha_1 = 3.41$ , to find the mle of  $\theta$ , we solve

$$\frac{\partial}{\partial \theta} \ln L = \frac{2\alpha}{\theta} - \sum_{x \neq \theta} \frac{\alpha + 1}{x_{x} + \theta} = \frac{2(3.41)}{\theta} - \frac{4.41}{18.7 + \theta} - \frac{4.41}{51.5 + \theta} = 0$$

 $\frac{\partial}{\partial \theta} \ln L = \frac{2\alpha}{\theta} - \sum \frac{\alpha+1}{x_i+\theta} = \frac{2(3.41)}{\theta} - \frac{4.41}{18.7+\theta} - \frac{4.41}{51.5+\theta} = 0 \ .$  This expression becomes  $\frac{6.82(18.7+\theta)(51.5+\theta) - 4.41\theta(51.5+\theta) - 4.41\theta(18.7+\theta)}{\theta(18.7+\theta)(51.5+\theta)} = 0 \ .$ 

The numerator is  $6568.001 + 169.182\theta - 2\theta^2$ .

Setting this equal to 0 results in two solutions for  $\theta$ , 113.5 and -28.9.

We ignore the negative root and use  $\theta_2 = 113.5$  as the new estimate for  $\theta$ .

The next estimate of  $\alpha$  is found from

$$\frac{2}{\alpha} + 2 \ln 113.5 - (\ln 132.2 + \ln 165.0) = 0.$$

Solving for  $\alpha$  results in  $\alpha_2 = 3.80$ .