EXAM FM QUESTIONS OF THE WEEK

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Week of August 7/06

A loan is amortized over 10 years with level monthly payments at the end of each month. The amount of principal paid in the final 12 payments is 2.5 times as large as the amount of principal repaid in the first 12 payments. Find the ratio $\frac{\text{amount of interest paid in final 12 payments}}{\text{amount of interest paid in first 12 payments}}$.

The solution can be found below.

Week of August 7/06 - Solution

Suppose that the loan amount is L and the monthly interest rate is j.

The monthly payment is $\frac{L}{a_{\overline{120}|j}} = K$.

The principal paid in the first 12 payments is

$K[v^{120} + v^{119} + \cdots + v^{109}]$,

and the amount of principal paid in the final 12 payments is

$K[v^{12} + v^{11} + \cdots + v^{1}]$.

We are told that $K[v^{12} + v^{11} + \cdots + v^{1}] = 2.5K[v^{120} + v^{119} + \cdots + v^{109}]$.

Therefore, $(1+j)^{108} = 2.5$, and $j = 2.5^{1/108} - 1 = .008520$.

The amount of interest paid in the first 12 payments is

$12K - K[v^{120} + v^{119} + \cdots + v^{109}] = K[12 - (a_{\overline{120}|j} - a_{\overline{108}|j})]$,

and the amount of interest paid in the final 12 payments is

$12K - K[v^{12} + v^{11} + \cdots + v^{1}] = K[12 - a_{\overline{12}|j}]$.

The ratio of interest in final 12 payments over interest in first 12 payments is

$$\frac{K[12-a_{\overline{12}|j}]}{K[12-(a_{\overline{120}|j}-a_{\overline{108}|j})]} = \frac{12-a_{\overline{12}|j}}{12-(a_{\overline{120}|j}-a_{\overline{108}|j})} = \frac{12-11.3610}{12-4.5445} = .0857 .$$