EXAM C QUESTIONS OF THE WEEK

S. Broverman, 2005

Question 3 - Week of August 8

You are given the following times of death and censoring (+) for a group of 10 individuals under observation from time 0:

$$1 \, , \, 2 \, , \, 4 \, , \, 4^+ \, , \, 5 \, , \, 6^+ \, , \, 8 \, , \, 8^+ \, , \, 11 \, , \, 12$$

(a) Find the Product-Limit estimate $S_{10}(t)$ and Nelson-Aalen estimate $\widehat{H}(t)$ for $0 \le t \le 12$.

(b) Suppose that there is an additional observation truncated at time 3 (comes under observation at time 3) and right-censored at time $c \ge 4$, where $t_i \le c < t_{i+1}$ (two consecutive death times). You are given that the Product-Limit estimate of $S(11)$ is .2286 . Find t_i and t_{i+1}.

Question 3 Solution

(a) $S_{10}(1) = \frac{9}{10} = .9$, $S_{10}(2) = \frac{9}{10} \cdot \frac{8}{9} = \frac{8}{10} = .8$, $S_{10}(4) = \frac{8}{10} \cdot \frac{7}{8} = \frac{7}{10} = .7$,

$S_{10}(5) = \frac{7}{10} \cdot \frac{5}{6} = \frac{7}{12} = .5833$, $S_{10}(8) = \frac{7}{12} \cdot \frac{3}{4} = \frac{7}{16} = .4375$,

$S_{10}(11) = \frac{7}{16} \cdot \frac{1}{2} = \frac{7}{32} = .21875$, $S_{10}(12) = \frac{7}{32} \cdot 0 = 0$.

$\widehat{H}(1) = \frac{1}{10} = .1$, $\widehat{H}(2) = \frac{1}{10} + \frac{1}{9} = .2111$, $\widehat{H}(4) = \frac{1}{10} + \frac{1}{9} + \frac{1}{8} = .3361$,

$\widehat{H}(5) = \frac{1}{10} + \frac{1}{9} + \frac{1}{8} + \frac{1}{6} = .5028$, $\widehat{H}(8) = \frac{1}{10} + \frac{1}{9} + \frac{1}{8} + \frac{1}{6} + \frac{1}{4} = .7528$,

$\widehat{H}(11) = \frac{1}{10} + \frac{1}{9} + \frac{1}{8} + \frac{1}{6} + \frac{1}{4} + \frac{1}{2} = 1.2528$,

$\widehat{H}(12) = \frac{1}{10} + \frac{1}{9} + \frac{1}{8} + \frac{1}{6} + \frac{1}{4} + \frac{1}{2} + \frac{1}{1} = 2.2528$.

(b) If $4 \le c < 5$ then $S_{11}(11) = \frac{9}{10} \cdot \frac{8}{9} \cdot \frac{8}{9} \cdot \frac{5}{6} \cdot \frac{3}{4} \cdot \frac{1}{2} = .2222$.

If $5 \le c < 8$ then $S_{11}(11) = \frac{9}{10} \cdot \frac{8}{9} \cdot \frac{8}{9} \cdot \frac{6}{7} \cdot \frac{3}{4} \cdot \frac{1}{2} = .2286$.

If $8 \le c < 11$ then $S_{11}(11) = \frac{9}{10} \cdot \frac{8}{9} \cdot \frac{8}{9} \cdot \frac{6}{7} \cdot \frac{4}{5} \cdot \frac{1}{2} = .2438$.

Therefore, the censoring occurs at or after time death time 5 but before death time 8.