EXAM M QUESTIONS OF THE WEEK

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Question 3 - Week of August 8

You are given the following for every integer age x:
(i) $\ddot{a}_x = 10$ and (ii) $A_x = \frac{11}{21}$

(a) Calculate ${}_{10}q_{50}$.

(b) Assuming UDD over each year of age, calculate $\overline{a}_{20:\overline{10}|}$.

The solution can be found below.

Question 3 Solution

(a) Using the relationship $A_x = 1 - d\ddot{a}_x$ we get $\frac{11}{21} = 1 - 10d$, from which it follows that $d = \frac{1}{21}$. Then $v = 1 - d = \frac{20}{21} = \frac{1}{1+i}$ and $i = \frac{1}{20} = .05$.

Then using the relationship $\ddot{a}_x = 1 + vp_x\ddot{a}_{x+1}$ we get $10 = 1 + (\frac{20}{21})(p_x)(10)$ from which it follows that $p_x = .945$. This is valid for any integer x.

Then, ${}_{10}p_{50} = p_{50} \cdot p_{51} \cdots p_{59} = (.945)(.945)\cdots(.945) = (.945)^{10} = .5680$, and ${}_{10}q_{50} = 1 - {}_{10}p_{50} = .4320$.

(b) We use the relationship $\overline{a}_x = \overline{a}_{x:\overline{n}|} + v^n {}_np_x\, \overline{a}_{x+n}$:

$$\overline{a}_{20} = \overline{a}_{20:\overline{10}|} + v^{10}\, {}_{10}p_{20}\, \overline{a}_{30}\ .$$

From UDD we have $\overline{A}_x = \frac{i}{\delta} A_x = \frac{.05}{ln\, 1.05} \cdot \frac{11}{21} = .5368$ for all x, and then $\overline{a}_x = \frac{1-\overline{A}_x}{\delta} = 9.494$ for all x.

Therefore $9.494 = \overline{a}_{20:\overline{10}|} + v^{10}\, {}_{10}p_x \cdot (9.494) = \overline{a}_{20:\overline{10}|} + \frac{.5680}{(1.05)^{10}} \cdot (9.494)$ and then $\overline{a}_{20:\overline{10}|} = 6.2$.

Note that since $v = \frac{20}{21}$ and $p_x = .945$, we have $vp_x = .9$, so that $v^{10}\, {}_{10}p_x = (.9)^{10} = .348678$.