

EXAM C QUESTIONS OF THE WEEK

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A portfolio of insurance policies consists of three types of policies. The distribution of the number of losses in one year for each type of policy is summarized as follows:

Policy Type	Type I	Type II	Type III
Annual Number of Losses	Poisson With Mean 1	Poisson With Mean 2	Poisson With Mean 4

Half of the policies are of Type I, one-quarter of the policies are of Type II and one-quarter are Type III.

A policy is chosen at random, and the number of losses in one year is 1.

Find the posterior probabilities that the policy type is I, II or III.

The solution can be found below.

Week of April 16/07 - Solution

$$P(\text{Type I}|X = 1) = \frac{P(X=1|\text{Type I}) \cdot P(\text{Type I})}{P(X=1)} .$$

$$\text{Type I, } \frac{1}{2}$$

$$\text{Type II, } \frac{1}{4}$$

$$\text{Type III, } \frac{1}{4}$$

$$P(X = 1|\text{Type I}) = e^{-1}$$

$$P(X = 1|\text{Type II}) = 2e^{-2}$$

$$P(X = 1|\text{Type III}) = 4e^{-4}$$

$$P(X = 1 \cap \text{Type I})$$

$$= \frac{1}{2}e^{-1}$$

$$P(X = 1 \cap \text{Type II})$$

$$= \frac{1}{2}e^{-2}$$

$$P(X = 1 \cap \text{Type III})$$

$$= e^{-4}$$

$$P(X = 1) = \frac{1}{2}e^{-1} + \frac{1}{2}e^{-2} + e^{-4}$$

$$P(\text{Type I}|X = 1) = \frac{\frac{1}{2}e^{-1}}{\frac{1}{2}e^{-1} + \frac{1}{2}e^{-2} + e^{-4}} = .681 ,$$

$$P(\text{Type II}|X = 1) = \frac{\frac{1}{2}e^{-2}}{\frac{1}{2}e^{-1} + \frac{1}{2}e^{-2} + e^{-4}} = .251 ,$$

$$P(\text{Type III}|X = 1) = \frac{e^{-4}}{\frac{1}{2}e^{-1} + \frac{1}{2}e^{-2} + e^{-4}} = .068 .$$