

EXAM C QUESTION OF THE WEEK

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Week of April 21/08

X and Y are both Poisson random variables.

Sample values of X and Y are drawn together in pairs (x_i, y_i) $z_i = x_i + y_i$.

The random variable Z is the sum of X and Y , and the z_i 's are the sample values of Z .

Limited fluctuation credibility is applied to Z based on the sample mean of Z being within 5% of the true mean of Z with probability 90%.

You are given that the expected number of sample values of Z needed for full credibility is 118.4, and the expected sum of the observed values of Z needed for full credibility is 947.1.

Find the covariance between X and Y .

The solution can be found below.

Week of April 21/08 - Solution

$$1082.4 \times \frac{\text{Var}(X+Y)}{[E(X+Y)]^2} = 118.4 \text{ and } 1082.4 \times \frac{\text{Var}(X+Y)}{E(X+Y)} = 947.1 .$$

It follows that $E(X + Y) = E(X) + E(Y) = 8.0$.

Since X and Y are Poisson, we have $\text{Var}(X) + \text{Var}(Y) = 8.0$.

But we also have $\text{Var}(X + Y) = 947.1 \times \frac{8}{1082.4} = 7.0$.

From $\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y) + 2\text{Cov}(X, Y)$,we get $7 = 8 + 2\text{Cov}(X, Y)$ so that $\text{Cov}(X, Y) = -\frac{1}{2}$.