EXAM C QUESTIONS OF THE WEEK

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Week of August 20/07

Annual aggregate losses S follow a compound distribution with annual frequency N
and severity X (the usual assumption of independence of N and the X's applies).
The probability function of N is uniform on the integers from 0 to 4.
X has a uniform distribution on the integers from 1 to 5 .

Annual stop loss insurance on aggregate losses has a deductible of 2. The insurer collects a
premium equal to the sum of the mean and standard deviation of the stop loss. Find the stop loss
premium.

The solution can be found below.

Week of August 20/07 - Solution

$E[N] = 2$, $Var[N] = 2$, $E[X] = 3$, $Var[X] = 2$.
$E[S] = E[N] \cdot E[X] = 6$.

The stop loss insurance pays $(S - 2)_+ = S - (S \wedge 2)$.
$E[S \wedge 2] = P(S = 1) + 2P(S > 1)$.
$P(S = 0) = P(N = 0) = .2$, $P(S = 1) = P(N = 1) \cdot P(X = 1) = (.2)(.2) = .04$.
$P(S > 1) = 1 - .2 - .04 = .76$.
$E[S \wedge 2] = .04 + 2(.76) = 1.56$.
$E[(S - 2)_+] = 6 - 1.56 = 4.44$.

$E[(S - 2)_+^2] = E[S^2] - E[(S \wedge 2)^2] - 2(2)[E(S) - E(S \wedge 2)]$.

$Var[S] = E[N] \cdot Var[X] + Var[N] \cdot (E[X])^2 = 22 = E[S^2] - (E[S])^2$.
$E[S^2] = 22 + 36 = 58$.
$E[(S \wedge 2)^2] = P(S = 1) + 4P(S > 1) = .04 + 4(.76) = 3.08$.
$E[(S - 2)_+^2] = 58 - 3.08 - 4(6 - 1.56) = 37.16$.
$Var[(S - 2)_+] = 37.16 - (4.44)^2 = 17.45$.

The premium for the stop loss insurance is $6 + \sqrt{17.45} = 10.18$.