

EXAM C QUESTIONS OF THE WEEK

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Week of February 4/08

S_1 has a compound distribution with frequency N and severity X_1
and S_2 has a compound distribution with frequency N and severity X_2 .

N is from the $(a, b, 0)$ class of distributions.

X_1 has an exponential distribution with mean θ , and the mean and variance of S_1
are 72 and 2268 .

X_2 has a uniform distribution on the interval $(0, \theta)$ (same value of θ as X_1), and the mean and
variance of S_2 are 36 and 351 .

Find $P(N = 0)$.

The solution can be found below.

Week of February 4/08 - Solution

$$E(S_1) = E(N) \times E(X_1) = E(N) \times \theta = 72 \quad (\text{Eq. 1})$$

$$\begin{aligned} \text{Var}(S_1) &= E(N) \times \text{Var}(X_1) + \text{Var}(N) \times [E(X_1)]^2 \\ &= [E(N) + \text{Var}(N)] \times \theta^2 = 2268 \quad (\text{Eq. 2}) \end{aligned}$$

$$E(S_2) = E(N) \times E(X_2) = E(N) \times \frac{\theta}{2} = 36 \quad (\text{Eq. 3})$$

$$\begin{aligned} \text{Var}(S_2) &= E(N) \times \text{Var}(X_2) + \text{Var}(N) \times [E(X_2)]^2 \\ &= E(N) \times \frac{\theta^2}{12} + \text{Var}(N) \times \frac{\theta^2}{4} = 351 \quad (\text{Eq. 4}) \end{aligned}$$

From Equations 2 and 4, we get $2 \times \text{Var}(N) \times \theta^2 = 1944$

so that $\text{Var}(N) \times \theta^2 = 972$, and then from equation 2 we have

$E(N) \times \theta^2 = 1296$. Now from Equation 1, we get

$$\theta = \frac{E(N) \times \theta^2}{E(N) \times \theta} = \frac{1296}{72} = 18.$$

From this we get $E(N) = 4$ and $\text{Var}(N) = 3$.

Since N is in the $(a, b, 0)$ class, it must be either Poisson, Negative Binomial or Binomial. Binomial is the only one of these with $\text{Var}(N) < E(N)$,

so N is binomial. If the parameters of N are m and q , then

$mq = 4$ and $mq(1 - q) = 3$, so that $1 - q = \frac{3}{4}$, and $q = \frac{1}{4}$ and $m = 16$.

Then $P(N = 0) = \binom{16}{0} q^0 (1 - q)^{16} - \left(\frac{3}{4}\right)^{16} = .0100$.