EXAM C QUESTIONS OF THE WEEK

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Week of February 5/07

An insurer has data for 8 independent policies. Each policy has a ground up loss distribution that is exponential. The policy information is as follows:

Policy	Payment	Deductible	Limit	Mean of Ground up Loss
1	10	None	None	θ
2	10	None	10	θ
3	10	5	None	θ
4	10	5	10	θ
5	10	None	None	2θ
6	10	None	10	2θ
7	10	5	None	3θ
8	10	5	10	3θ

Find the mle of θ.

The solution can be found below.

Week of February 5/07 - Solution

The likelihood function will be

$f_1(10) \cdot [1 - F_2(10)] \cdot \frac{f_3(15)}{1-F_3(5)} \cdot \frac{1-F_4(15)}{1-F_4(5)} \cdot f_5(10) \cdot [1 - F_6(10)] \cdot \frac{f_7(15)}{1-F_7(5)} \cdot \frac{1-F_8(15)}{1-F_8(5)}$.

For policy 3, the payment is 10 after a deductible of 5, so the ground up loss is 15. A similar comment applies to other factors in the likelihood. Also, for policy 4, the policy limit is $u - d$, where $u = 15$ is the maximum covered loss, so that the policy limit is $15 - 5 = 10$, but a limit payment is triggered by a ground up loss greater than 15.

This likelihood becomes

$L(\theta) = \frac{1}{\theta} e^{-10/\theta} \cdot e^{-10/\theta} \cdot \frac{\frac{1}{\theta} e^{-15/\theta}}{e^{-5/\theta}} \cdot \frac{e^{-15/\theta}}{e^{-5/\theta}} \cdot \frac{1}{2\theta} e^{-10/2\theta} \cdot e^{-10/2\theta} \cdot \frac{\frac{1}{3\theta} e^{-15/3\theta}}{e^{-5/3\theta}} \cdot \frac{e^{-15/3\theta}}{e^{-5/3\theta}}$

$= \frac{1}{6\theta^4} \frac{e^{-70/\theta}}{e^{-13.33/\theta}} = \frac{1}{6\theta^4} e^{-56.67/\theta}$. The mle of θ is $\widehat{\theta} = \frac{56.67}{4} = 14.17$.