

## EXAM C QUESTIONS OF THE WEEK

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### Week of January 14/08

A model for the lifetime of a tire assumes that a randomly chosen tire has a lifetime that is normally distributed with a mean of  $\Lambda$  miles and a standard deviation of  $a$  miles. According to the model,  $\Lambda$  is normally distributed with a mean of 80,000 and a standard deviation of 10,000. You are given that the 95th percentile of the lifetime of a randomly chosen tire is 116,783 miles. Find the probability that a randomly selected tire has a lifetime of at most 100,000 miles.

**The solution can be found below.**

## **Week of January 14/08 - Solution**

$\Lambda$  has a normal distribution with mean 80,000 and standard deviation 10,000

$X$  is the tire lifetime. We are told that the conditional distribution of  $X$  given  $\Lambda$  is normal with a mean of  $\Lambda$  and a standard deviation of  $a$ .

$X$  is a continuous mixture over  $\Lambda$ . The unconditional distribution of  $X$  is normal with a mean of 80,000 and a standard deviation of  $\sqrt{10,000^2 + a^2}$ .

The 95th percentile of  $X$  is  $80,000 + 1.645\sqrt{10,000^2 + a^2} = 116,783$ .

It follows that  $\sqrt{10,000^2 + a^2} = 22,360$ .

Then,  $P(X \leq 100,000) = P\left(\frac{X-80,000}{22,360} \leq \frac{100,000-80,000}{22,360}\right) = \Phi(.89) = .8133$ .