

EXAM C QUESTIONS OF THE WEEK

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Week of January 15/07

A population of people aged 50 consists of twice as many non-smokers as smokers. Non-smokers at age 50 have a mortality probability of .1 and smokers at age 50 have a mortality probability of .2. Two 50-year old individuals are chosen at random from the population.

- (a) Find the probability that at least one of them dies before age 51.
- (b) Suppose that the mortality probabilities for smokers and non-smokers remain the same at age 51. Find the mortality probability of a randomly chosen survivor at age 51 in this population.

The solution can be found below.

Week of January 15/07 - Solution

(a) For a randomly chosen individual at age 50, the mortality probability is the mixture of the mortality probabilities for non-smokers and smokers. This is $q = \left(\frac{2}{3}\right)(.1) + \left(\frac{1}{3}\right)(.2) = \frac{2}{15}$.

The probability that both of two independent 50-year old individuals survive the year is $\left(1 - \frac{2}{15}\right)^2 = .7511$. The probability at least one of them dies by age 51 is $1 - .7511 = .25$.

(b) Suppose that there are 1000 non-smokers and 500 smokers at age 50. The expected number of surviving non-smokers at age 51 is $1000(.9) = 900$, and the number of surviving smokers is $(500)(.8) = 400$. A randomly chosen survivor at age 51 has a $\frac{9}{13}$ chance of being a non-smoker and a $\frac{4}{13}$ chance of being a smoker. The mortality probability at age 51 for the randomly chosen survivor is $\left(\frac{9}{13}\right)(.1) + \left(\frac{4}{13}\right)(.2) = .131$.