EXAM C QUESTIONS OF THE WEEK

S. Broverman, 2007

Week of January 29/07

You are given that X has pdf $f(x) = \frac{4/\pi}{1+x^2}$ for $0 < x < \infty$.
How many of the following distributions have a lighter right tail than X?

I. Pareto with $\alpha = 1$ II. Pareto with $\alpha > 1$

III. Paralogistic with $\alpha = 1$ IV. Inverse paralogistic with $\tau > 1$

A) 0 B) 1 C) 2 D) 3 E) 4

The solution can be found below.

Week of January 29/07 - Solution

I. $\frac{f(x)}{f_I(x)} = \frac{4/\pi}{1+x^2} \cdot \frac{(x+\theta)^2}{\theta} \to \frac{4}{\pi\theta}$ as $x \to \infty$. Same right tail weight.

II. $\frac{f(x)}{f_{II}(x)} = \frac{4/\pi}{1+x^2} \cdot \frac{(x+\theta)^{\alpha+1}}{\alpha\theta^\alpha} = \frac{4}{\pi\alpha\theta^\alpha} \cdot \frac{(x+\theta)^2}{1+x^2} \cdot (x+\theta)^{\alpha-1} \to \infty$ as $x \to \infty$.
X has heavier right tail weight.

III. $\frac{f(x)}{f_{III}(x)} = \frac{4/\pi}{1+x^2} \cdot \theta[1+(x/\theta)]^2 = \frac{4\theta}{\pi} \cdot \frac{[1+(x/\theta)]^2}{1+x^2} \to \frac{4\theta}{\pi}$ as $x \to \infty$. Same right tail weight.

IV. $\frac{f(x)}{f_{IV}(x)} = \frac{4/\pi}{1+x^2} \cdot \frac{x[1+(x/\theta)^\tau]^{\tau+1}}{\tau^2(x/\theta)^\tau} = \frac{4}{\pi\tau^2} \cdot \frac{x[1+(x/\theta)^\tau]^{\tau+1}}{1+x^2} \to \infty$ as $x \to \infty$ since $\tau > 1$. X has heavier right tail weight. Answer: C