EXAM C QUESTIONS OF THE WEEK

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Week of July 16/07

An annual claim frequency model for a portfolio of insurance policies has the following characteristics.

- half of the policies have annual frequency that is Poisson distribution with a mean of 2
- half of the policies have annual frequency that is Poisson with a mean of 4
- the annual claims are independent from one year to the next conditional on the Poisson mean being known.

A policy is chosen at random from the portfolio and this policy results in an annual claim frequency of 2. Find the conditional mean of the annual claim frequency in the second year of the same policy.

The solution can be found below.

Week of July 16/07 - Solution

We wish to find $E[N_2|N_1=2]$, where N is a mixture of three Poisson distributions.

The conditional probability function of N_2 given $N_1=2$ is
$P[N_2=k|N_1=2]=\frac{P[N_2=k\cap N_1=2]}{P[N_1=2]}$.

The denominator is $P[N_1=2]=P[N_1=2|\lambda=2]\cdot P(\lambda=2)+P[N_1=2|\lambda=4]\cdot P(\lambda=4)$
$=(.5)[\frac{e^{-2}2^2}{2!}+\frac{e^{-4}4^2}{2!}]$.

The numerator is

$$P[N_2=k\cap N_1=2]=P[N_2=k\cap N_1=2|\lambda=2]\cdot P(\lambda=2)$$
$$+P[N_2=k\cap N_1=2|\lambda=4]\cdot P(\lambda=4)$$
$$=P[N_2=k|\lambda=2]\cdot P[N_1=2|\lambda=2]\cdot P(\lambda=2)$$
$$+P[N_2=k|\lambda=4]\cdot P[N_1=2|\lambda=4]\cdot P(\lambda=4)$$
$$=(.5)[P[N_2=k|\lambda=2]\cdot\frac{e^{-2}2^2}{2!}+P[N_2=k|\lambda=4]\cdot\frac{e^{-4}4^2}{2!}]$$

Then, $P[N_2=k|N_1=2]=\frac{P[N_2=k|\lambda=2]\cdot\frac{e^{-2}2^2}{2!}+P[N_2=k|\lambda=4]\cdot\frac{e^{-4}4^2}{2!}}{\frac{e^{-2}2^2}{2!}+\frac{e^{-4}4^2}{2!}}$

and

$$E[N_2|N_1=2]=\sum_{k=0}^{\infty}k\cdot P[N_2=k|N_1=2]=\sum_{k=0}^{\infty}k\cdot\frac{P[N_2=k|\lambda=2]\cdot\frac{e^{-2}2^2}{2!}+P[N_2=k|\lambda=4]\cdot\frac{e^{-4}4^2}{2!}}{\frac{e^{-2}2^2}{2!}+\frac{e^{-4}4^2}{2!}}$$

$$\sum_{k=0}^{\infty}k\cdot P[N_2=k|\lambda=2]\cdot\frac{\frac{e^{-2}2^2}{2!}}{\frac{e^{-2}2^2}{2!}+\frac{e^{-4}4^2}{2!}}+\sum_{k=0}^{\infty}k\cdot P[N_2=k|\lambda=4]\cdot\frac{\frac{e^{-4}4^2}{2!}}{\frac{e^{-2}2^2}{2!}+\frac{e^{-4}4^2}{2!}}$$

$$=(2)\cdot\frac{\frac{e^{-2}2^2}{2!}}{\frac{e^{-2}2^2}{2!}+\frac{e^{-4}4^2}{2!}}+(4)\cdot\frac{\frac{e^{-4}4^2}{2!}}{\frac{e^{-2}2^2}{2!}+\frac{e^{-4}4^2}{2!}}=2.70\ .$$

The problem could also have been solved using the rule
$E[N_2|N_1=k]=E[N_2|\lambda=2]\cdot P[\lambda=2|N_1=2]+E[N_2|\lambda=4]\cdot P[\lambda=4|N_1=2]$,
which results in the same expression, $(2)\cdot\frac{\frac{e^{-2}2^2}{2!}}{\frac{e^{-2}2^2}{2!}+\frac{e^{-4}4^2}{2!}}+(4)\cdot\frac{\frac{e^{-4}4^2}{2!}}{\frac{e^{-2}2^2}{2!}+\frac{e^{-4}4^2}{2!}}$.