EXAM C QUESTION OF THE WEEK

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Week of March 10/08

A data set of 200 loss amounts from a loss distribution is in interval grouped form based on the following four intervals:

$$(0, 100] \qquad (100, 300] \qquad (300, 500] \qquad (500, 900]$$

You are given the following information:

- The empirical estimate of the mean loss is 212.5.
- The empirical estimate of the limited expected loss with policy limit 200 is 125.
- The empirical estimate of the expected cost per payment with an ordinary deductible of 200 is 218.75.

Find the empirical estimate of the standard deviation of the loss.

The solution can be found below.

Week of March 10/08 - Solution

Suppose that there are n_1 losses in $(0, 100]$, n_2 losses in $(100, 300]$, n_3 losses in $(300, 500]$, and n_4 losses in $(500, 900]$.

We get 4 equations in n_1, n_2, n_3, n_4 .

(1) $n_1 + n_2 + n_3 + n_4 = 200$

(2) $\frac{50n_1+200n_2+400n_3+700n_4}{200} = 212.5$

$\rightarrow 50n_1 + 200n_2 + 400n_3 + 700n_4 = 42,500$

(3) $\frac{50n_1+150\cdot\frac{n_2}{2}+200(\frac{n_2}{2}+n_3+n_4)}{200} = 125$

$\rightarrow 50n_1 + 150 \cdot \frac{n_2}{2} + 200(\frac{n_2}{2} + n_3 + n_4) = 25,000$

(4) From $E[X - 200|X > 200] = \frac{E(X)-E(X\wedge 200)}{P(X>200)}$

we get the empirical estimate

$$\frac{212.5-125}{\frac{\frac{n_2}{2}+n_3+n_4}{200}} = 218.75 \rightarrow \frac{n_2}{2} + n_3 + n_4 = 80$$

From Equations (3) and (4) we get $50n_1 + 75n_2 + 200(80) = 25,000$
so that $50n_1 + 75n_2 = 9,000$.
From Equations (1) and (4) we get $n_1 + n_2 + (80 - \frac{n_2}{2}) = 200$
so that $n_1 + \frac{n_2}{2} = 120$.
From these two equations we get $n_1 = 90$ and $n_2 = 60$.
Combined with Equations (1) and (2) this gives us $90 + 60 + n_3 + n_4 = 200$
so that $n_3 + n_4 = 50$
and $(50)(90) + (200)(60) + 400n_3 + 700n_4 = 42,500$
so that $400n_3 + 700n_4 = 26,000.$
Then solving these equations results in $n_3 = 30$ and $n_4 = 20$.

The empirical estimate of $Var[X[$ is the estimated second moment minus the square of the estimated first moment. The estimate second moment is
$\frac{(90)(50^2)+(60)(200^2)+(30)(400^2)+(20)(700^2)}{200} = 86,125$.
The estimated loss variance is $86,125 - 212.5^2 = 40,968.75$.
The estimated loss standard deviation is $\sqrt{40.968.75} = 202.4$.