EXAM C QUESTIONS OF THE WEEK

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Week of March 19/07

You are given the following:
- The random variable X has the density function $f(x) = \frac{1}{\lambda}e^{-x/\lambda}$, $0 < x < \infty$, $\lambda > 0$.
- λ is estimated by an estimator $\tilde{\lambda}$ based on a large random sample of size n.
- p is the proportion of the observations in the sample that are greater than 1.
- The probability that X is greater than 1 is estimated by the estimator $e^{-1/\tilde{\lambda}}$.

(a) Determine the estimator for the probability that X is greater than 1 if $\tilde{\lambda}$ is the maximum likelihood estimator.

A) $\bar{X}$ B) $e^{-1/\bar{X}}$ C) p D) $-\,ln\,p$ E) $-\frac{1}{ln\,p}$

(b) Determine the approximate variance of the estimator for the probability that X is greater than 1 if $\tilde{\lambda}$ is $\bar{X}$.

A) $\frac{\lambda^2}{n}$ B) $\frac{1}{n}e^{-1/\lambda}$ C) $\frac{1}{n\lambda}e^{-1/\lambda}$ D) $\frac{1}{n\lambda^2}e^{-2/\lambda}$ E) $\frac{1}{n}e^{-1/\lambda}(1 - e^{-1/\lambda})$

The solution can be found below.

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(a) X has an exponential distribution. The mle is the same as the moment estimator of λ, which is $\overline{X}$. The estimate of $P[X > 1]$ is the estimate of $e^{-1/\lambda}$, which is found using the mle of λ. The estimate of $P[X > 1]$ is of $e^{-1/\tilde{\lambda}} = e^{-1/\bar{X}}$. Answer: B

(b) The variance of a function of the mle of parameter θ is $Var[g(\widehat{\theta})] = [g'(\theta)]^2 \cdot Var[\widehat{\theta}]$.
In this case, $g(\lambda) = P[X > 1] = e^{-1/\lambda} \rightarrow g'(\lambda) = e^{-1/\lambda} \cdot \frac{1}{\lambda^2}$. Since the mle of λ in an exponential distribution is $\widehat{\lambda} = \overline{X}$, $Var[\widehat{\lambda}] = Var[\overline{X}] = \frac{Var[X]}{n} = \frac{\lambda^2}{n}$. The variance of the estimate of $P[X > 1]$ is $(e^{-1/\lambda} \cdot \frac{1}{\lambda^2})^2 \cdot \frac{\lambda^2}{n} = \frac{1}{n\lambda^2} e^{-2/\lambda}$. Answer: D