EXAM C QUESTION OF THE WEEK

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Week of March 31/08

Loss random variable X has a uniform distribution on $(0, \theta)$.

A sample is taken of n insurance payments from policies with a limit of 100.
Eight of the sample values are limit payments of 100.
The maximum likelihood estimate of θ is $\widehat{\theta}$.

Another sample is taken, also of n insurance payments, but from policies with a limit of 150.
Three of the sample values are limit payments of 150.
The maximum likelihood estimate of θ is $\frac{4}{3}\widehat{\theta}$.

Determine n.

The solution can be found below.

Week of March 31/08 - Solution

Suppose that m of the sample values are limit values.

The likelihood function for the first estimation is $\left(\frac{1}{\theta}\right)^{n-m}\left(\frac{\theta-100}{\theta}\right)^m = \frac{(\theta-100)^m}{\theta^n}$.

The log of the likelihood is $\ell(\theta) = m\,ln(\theta-100) - n\,ln\,\theta$.

For the first sample we have

$\frac{\partial}{\partial\theta}\,\ell(\theta) = \frac{8}{\theta-100} - \frac{n}{\theta} = 0$, so that the mle of θ is $\widehat{\theta} = \frac{100n}{n-8}$.

For the second sample we have

$\frac{\partial}{\partial\theta}\,\ell(\theta) = \frac{3}{\theta-150} - \frac{n}{\theta} = 0$, so that the mle of θ is $\frac{150n}{n-3}$.

We are given that $\frac{150n}{n-3} = \frac{4}{3} \cdot \frac{100n}{n-8}$.

Solving for n results in $n = 48$.