EXAM C QUESTIONS OF THE WEEK

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Week of September 24/07

Maximum likelihood estimation is being applied to estimate the mean of an exponential loss distribution. The sample data is based on insurance payments with no deductible, but with a common policy limit. The data set consists of 50 losses that are below the policy limit.
The MLE is 3.00 . If an additional limit payment is added to the data set the MLE becomes 3.08. Suppose that an additional insurance payment of 3.5 is added to the original data. Find the new MLE in this case.

The solution can be found below.

Week of September 24/07 - Solution

For maximum likelihood estimation applied to the exponential distribution with right-censoring at u, if a data set has k uncensored data points (say $x_1, ..., x_k$) and m data points censored at u, the MLE of the exponential mean is $\dfrac{\sum\limits_{i=1}^{k} x_i + mu}{k}$ (the denominator is the number of uncensored observations). We are given the following:

$$\frac{\sum\limits_{i=1}^{50} x_i + mu}{50} = 3 \quad \text{and} \quad \frac{\sum\limits_{i=1}^{50} x_i + (m+1)u}{50} = 3.08$$

From this we see that $u = 4$.

Therefore, the insurance payment of 3.5 is below the policy limit, so if it is added to the original data set, the new MLE is $\dfrac{\sum\limits_{i=1}^{51} x_i + mu}{51} = \dfrac{\sum\limits_{i=1}^{50} x_i + 3.5 + mu}{51}$.

From $\dfrac{\sum\limits_{i=1}^{50} x_i + mu}{50} = 3$ we know that $\sum\limits_{i=1}^{50} x_i + mu = 150$.

Therefore, $\dfrac{\sum\limits_{i=1}^{50} x_i + 3.5 + mu}{51} = \dfrac{150 + 3.5}{51} = 3.01$ is the new MLE.