

# EXAM M QUESTIONS OF THE WEEK

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## **Week of December 12**

A portfolio of insurance policies is made up of non-smokers and smokers. Non-smokers make up 75% of the policyholders. The model being used for mortality is that non-smokers have a constant force of mortality of .01 and smokers have a constant force of mortality of .02.

A policyholder is chosen at random from the group, and it is found that the policyholder is still alive 20 years after the policy is issued. Find the probability that the policyholder is a smoker.

**The solution can be found below.**

## **Week of December 12 - Solution**

Let  $T$  represent the exact time until death of the randomly chosen policyholder.

We wish to find  $P[\text{Smoker}|T > 20] = \frac{P[\text{Smoker} \cap (T > 20)]}{P[T > 20]}$ .

$$P[\text{Smoker} \cap (T > 20)] = P[(T > 20)|\text{Smoker}] \cdot P[\text{Smoker}] = e^{-20(.02)} \cdot (.25) = .167580.$$

$$P[T > 20] = P[(T > 20) \cap \text{Smoker}] + P[(T > 20) \cap \text{Non-smoker}]$$

$$= P[(T > 20)|\text{Smoker}] \cdot P[\text{Smoker}] + P[(T > 20)|\text{Non-smoker}] \cdot P[\text{Non-smoker}]$$

$$= e^{-20(.02)} \cdot (.25) + e^{-20(.01)} \cdot (.75) = .781628.$$

$$\text{Then } P[\text{Smoker}|T > 20] = \frac{P[\text{Smoker} \cap (T > 20)]}{P[T > 20]} = \frac{.167580}{.781628} = .214.$$