EXAM P QUESTIONS OF THE WEEK

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Week of December 5/05

Fred, Ned and Ted each have season tickets to the Toronto Rock (Lacrosse).
Each one of them might, or might not attend any particular game. The probabilities
describing their attendance for any particular game are
$P[$at least one of them attends the game$] = .95$,
$P[$at least two of them attend the game$] = .80$, and
$P[$all three of them attend the game$] = .50$.
Their attendance pattern is also symmetric in the following way
$P(F) = P(N) = P(T)$ and $P(F \cap N) = P(F \cap T) = P(N \cap T)$,
where F, N and T denote the events that Fred, Ted and Ned attended the game, respectively.

For a particular game, find the probability that Fred and Ned attended.
A) .15   B) .30   C) .45   D) .60   E) .75

The solution can be found below.

Week of December 5 - Solution

From the given probabilities, it follows that

$P[$exactly two of them attend the game$]$
$= P[$at least two of them attend the game$] - P[$all three of them attend the game$]$
$= .80 - .50 = .30$.

We also know that
$P[$exactly two of them attend the game$]$
$= P(F \cap N \cap T') + P(F \cap N' \cap T) + P(F' \cap N \cap T) = .3$.

From the symmetry of the probabilities, the three on the right hand side of the equation are equal,
so that $P(F \cap N \cap T') = P(F \cap N' \cap T) = P(F' \cap N \cap T) = .1$.
They are equal because
$P(F \cap N \cap T') = P(F \cap N) - P(F \cap N \cap T)$ and
$P(F \cap T \cap N') = P(F \cap T) - P(F \cap N \cap T)$ and
$P(N \cap T \cap F') = P(N \cap T) - P(F \cap N \cap T)$, which are all equal.

The probability that Fred and Ned attend is then
$P(F \cap N) = P(F \cap N \cap T') + P(F \cap N \cap T) = .1 + .5 = .6$.