ACTEX EXAM C STUDY MANUAL – 2007 Edition

Errata List, by S. Broverman
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Mar 6/07 Page LM-90, #7 solution, line 4 should be

$$E[Y^2] = \left(\frac{2}{3}\right)(2\times 1^2) + \left(\frac{1}{3}\right)(2\times 2^2) = 4 \to Var[Y] = 4 - \left(\frac{4}{3}\right)^2 = \frac{20}{9}$$

Feb 5/07 Page LM-97, Example LM-38, should just ask for the mean of the spliced distribution

Feb 21/07 Page LM-111, Example LM-44 solution, last 4 lines should be
The expected gain by the game operator for one play of the game is $P-8=1$, so that $P=9$.
A player has a net gain if $N^2 > 9$, which is equivalent to $N \ge 4$. The probability of a player having a net gain on one play of the game is $P(N \ge 4) = 1 - P(N = 0,1,2,3)$

$$= 1 - \left[\frac{1}{0!(1+1)^2} + \frac{2}{1!(1+1)^{2+1}} + \frac{2(3)}{2!(1+1)^{2+2}} + \frac{2(3)(4)}{3!(1+1)^{2+3}}\right] = \frac{3}{16}$$

Jan 19/07 Page LM-112, Example LM-44 solution, last line
The answer should be 60.96 .

Jan 19/07 Page LM-113, Example LM-45 solution, last two lines should be

Then $S_X(x) = E[S_{X|\Lambda}(x\,|\,\lambda)] = \int_1^2 \frac{1}{(1+x)^\lambda}\cdot 1\,d\lambda = \left.\frac{-1}{(1+x)^\lambda \ln(1+x)}\right|_{\lambda=1}^{\lambda=2}$

$$= \frac{1}{\ln(1+x)}\cdot\left[\frac{1}{1+x} - \frac{1}{(1+x)^2}\right] = \frac{x}{(1+x)^2\ln(1+x)} \quad \text{for } x > 0 .$$

Jan 16/07 Page LM-118, #5 solution, last two lines should be

The 95-th percentile of X is c, where $P(X \le c) = P\left(\frac{X-1}{\sqrt{2}} \le \frac{c-1}{\sqrt{2}}\right) = \Phi\left(\frac{c-1}{\sqrt{2}}\right) = .95$.

From the standard normal table we get $\frac{c-1}{\sqrt{2}} = 1.645$, so that $c = 3.33$.

Jan 28/07 Page LM-130, Example LM-49(b) solution, 2nd line

$\frac{\theta^2}{2}$ should be $\frac{(\theta-d)^3}{3\theta}$ in both places

Jan 30/07 Page LM-148, Example LM-54 solution, last five lines should be

$$= e^{9.6192} \cdot \Phi\left(-3.47\right) + 20^2\left[1-\Phi(-2.19)\right] = 398.8 .$$

Then, $E[Y_L^2] = 15,051 - 398.8 - 40[9.96 - 19.94] = 11,451$,

and $Var[Y_L] = 11,451 - 80.0^2 = 5051$.

We find $E[Y_P^2]$ from $E[Y_P^2] = \dfrac{E[Y_L^2]}{1-F_X(20)} = \dfrac{11,451}{1-\Phi\left(\dfrac{\ln 20 - \mu}{\sigma}\right)} = 11,617$,

and then $Var[Y_P] = 11,617 - (81.16)^2 = 5030$.

Jan 31/07 Page LM-153, #5 solution, 2nd line, the pdf should be .000025

Jan 31/07 Page LM-154, #5 solution, final two lines should be

$$E[Y_P^2] = \frac{E[Y_L^2]}{P(X > 2000)} = \frac{225,000}{.075} = 3,000,000 \ .$$

$$Var[Y_P] = 3,000,000 = 1500^2 = 750,000 .$$

Feb 7/07 Page LM-172, Equations 14.3 and 14.5
$F_X(d)$ should be $F_X(u)$.

Feb 26/07 Page LM-172, Example LM-57 solution, last line
$F(5000)$ should be $F(25,000)$.

Feb 1/07 Page LM-215, Example LM-69, solution
In line 3, the variance of N is $Var[N] = 1.5 - 1 = .5$.
In lines 4 and 6, 5000/3 should be 2500/3 .
In line 6, 8750/3 should be 6250/3 .

Mar 1/07 Page LM-267, #2 solution
In line 9, the second moment should be $2(100^2) = 20,000$.
Also, 10,000 should be 20,000 in lines 10 and 11 and 8187.2 should be 16,374.6 in lines 10 and 11. A similar comment applies to lines 16 and 17. The variance of S should be 327,492.

Feb 27/07 Page LM-290, #7 answers should
A) Less than .26 B) At least .26 but less than .27 C) B) At least .28 but less than .28
D) At least .28 but less than .29 E) At least .29

Apr 4/07 Page ME-11, Example ME-4, Solution, line 1, denominator should be 60 (not 50)

Apr 4/07 Page ME-15, Example ME-5, the alternative hypothesis should be $H_1 : \mu_X \neq 5.0$
The solution should be changed as follows:
First line should have $\overline{X} = \frac{330}{6} = 5.5$
2nd line should have 5.5-5.0 in numerator (not 6.5-5.0)
.74 should be changed to .247 in the rest of the solution.
$\Phi(.247) = .5975$
In the 2nd last line, .23 should be .4025 and .46 should be .805

Mar 3/07 Page ME-48, #2 solution line 3, 7/108 should be 5/108

Mar 10/07 Page ME-92, #10, the interval should be (2.71,3.10)

Mar 29/07 Page ME-101, #20 solution, line 10, $S_{80}(3)$ should be $S_{80}(2)$

Apr 9/07 Page ME-111, Loglogistic example, line 6 should be $\left(\frac{21}{\theta}\right)^{\gamma} = 3$
This changes the value of γ to 1.53 and the value of θ to 10.2.
The probability $P[X > 10]$ is .51

Mar 21/07 Page ME-126, 3rd paragraph, line 4, should say "does not involve the parameter …"

Mar 216/07 Page ME-153, #2 solution, line 4, the denominator should be 9, and the answer should be 31.56

Mar 27/07 Page ME-166, #6 line 3, α should be θ

Mar 21/07 Page ME-196, solution to #10 should be
If $\hat{\theta}$ is an estimator of the parameter θ, the mean square error of $\hat{\theta}$ is
$MSE(\hat{\theta}) = E[(\theta - \hat{\theta})^2]$. The variance of $\hat{\theta}$ is $Var(\hat{\theta}) = E[(\theta - E(\hat{\theta}))^2]$.
These will be the same if $E(\hat{\theta}) = \theta$, which is the same as saying that $\hat{\theta}$ is an
unbiased estimator. Answer: D

Jan 19/07 Page CR-39, #13, first line should be
13. A portfolio of insureds consists of two types of insureds. Losses from the two types are:

Feb 3/07 Page CR-106, 3rd line from top of page, at the end of the line should be
$M = \max\{x_1, ..., x_n, \theta\}$

Feb 6/07 Page CR-108, A variation on the Beta prior and Negative Binomial model distributions
The entire section should be replaced with the following.
The parametrization of the negative binomial distribution in the Exam C table has parameters *r* and β, both > 0. A variation on this parametrization is to keep the parameter *r*, but use the parameter *q*, where $q = \frac{1}{1+\beta}$ so that $0 < q < 1$. The probability function of the negative binomial for $k = 0,1,2,...$ is

$$p_k = \binom{k+r-1}{k}\frac{\beta^k}{(1+\beta)^{r+k}} = \frac{r(r+1)\cdots(r+k-1)}{k!}\left(\frac{1}{1+\beta}\right)^r\left(\frac{\beta}{1+\beta}\right)^k = \frac{\Gamma(r+k)}{\Gamma(r)\Gamma(k+1)}q^r(1-q)^k.$$

Suppose that we use this parametrization, and that we assume that *q* is a prior parameter with a beta distribution with parameters *a* and *b*, with prior density $\pi(q) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}q^{a-1}(1-q)^{b-1}$.
Assume that the conditional distribution of *X* given *q* is negative binomial with known parameter *r*, and with parameter *q*, so that the model distribution has probability function

$$f(x\,|\,q) = \frac{\Gamma(r+x)}{\Gamma(r)\Gamma(x+1)}q^r(1-q)^x \quad \text{for} \quad x = 0,1,2,... \quad .$$

The joint distribution of *X* and *q* has joint density

$$f(x,q) = f(x\,|\,q)\pi(q) = \frac{\Gamma(r+x)}{\Gamma(r)\Gamma(x+1)}q^r(1-q)^x \cdot \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}q^{a-1}(1-q)^{b-1}$$

$$= \frac{\Gamma(r+x)}{\Gamma(r)\Gamma(x+1)} \cdot \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}q^{a+r-1}(1-q)^{b+x-1}.$$

This is proportional, in *q*, to $q^{a+r-1}(1-q)^{b+x-1}$, which implies that the posterior distribution of also has a beta distribution, with parameters $a' = a+r$ and $b' = b+x$.

If there are observations of *x* available, say $x_1, x_2, ..., x_n$, then the joint density of $x_1, x_2, ..., x_n, q$ is proportional to $q^{a+nr-1}(1-q)^{b+\Sigma x_i-1}$. Again this implies that the posterior distribution of *q* is beta, with parameters $a' = a+nr$ and $b' = b+\Sigma x_i$.

Under this parametrization, the beta distribution is a conjugate prior for the negative binomial.

Apr 23/07 Page CR-180,181, #19 solution

In the last two lines of page 190, the 2nd moment should have 3 instead of 2 in the denominator, and the 2nd empirical moment should be 155,333.

The top of page 191 should be

The empirical estimate of the variance of *X* is 106,933.

In the semiparametric empirical Bayes credibility model, we use the empirical estimate of $E[X^2]$ for μ, so that $\hat{\mu} = 220$. We also know that $Var[X] = v + a$, so using the empirical estimate of $Var[X]$ gives $106,933 = \hat{v} + \hat{a}$. But we also know that, for this model, $\hat{v} - \hat{a} = \mu^2$, so using our sample estimate of μ, we have $\hat{v} - \hat{a} = 220^2$. We can then solve the two equations $106,933 = \hat{v} + \hat{a}$ and $\hat{v} - \hat{a} = 220^2$ to get $\hat{v} = 77,667$ and $\hat{a} = 29,267$.

We can now find the estimated loss in the 3rd year for a policy that had losses of $Y_1 = 150$ in the first year and $Y_2 = 0$ in the second year. The estimate is $\hat{Z}\overline{Y} + (1-\hat{Z})\hat{\mu}$,

where $\hat{Z} = \dfrac{2}{2 + \dfrac{\hat{v}}{\hat{a}}} = .4298$ and $\hat{\mu} = 220$. The credibility premium is

(.4298)(75)+(.5702)(220) = 158.

Apr 4/07 Page SI-8, 2nd last line, $\dfrac{n - P_n}{P_n}$ should be $\dfrac{1 - P_n}{P_n}$

Mar 13/07 Page SI-43, Line 1, second sentence should say

If the *X*'s are all normal random variables and are either independent or have a multivariate normal distribution, then the sum of the *X*'s will also have a normal distribution.

Apr 10/07 Page SI-46, Example SI-12(c), 4th line, b should be 221.62,

And in the 6th line the interval should be (70.90,221.62).

Apr 4/07 Page SI-46, $E[S_t \mid S_t < K]$, in the middle equation, the numerator should be

$$\Phi\left(\frac{\ln K - \ln S_0 - (\alpha - \delta + \frac{1}{2}\sigma^2)t}{\sigma\sqrt{t}}\right)$$

(the equation at the right on that line is correct.

$E[S_t \mid S_t > K]$, the denominator should be $\Phi\left(\dfrac{\ln S_0 - \ln K + (\alpha - \delta - \frac{1}{2}\sigma^2)t}{\sigma\sqrt{t}}\right)$

Example SI-12, $E[S_1 \mid S_1 > 100]$, the denominator should be

$$\Phi\left(\frac{\ln 100 - \ln 100 + (\ln 1.1 - \frac{1}{2}(.04))}{.2}\right)$$

and the answer should be

$$110 \cdot \frac{\Phi(.58)}{\Phi(.38)} = 110 \cdot \frac{.7190}{.6480} = 122.05$$

Apr 4/07 Page SI-47, Example SI-12 (continued), d_2 should be $-.27$,
The call option price should be 12.5,3and the put option price should be 15.82

Apr 4/07 Page SI-47, 3rd line from bottom,
… data quantile ..” should be “… data point ..”

May 11/07 Page SI-48, Example SI-13, solution
Some of the quantiles are incorrect.
.835 should be 1.036 , .575 should be .674 and .136 should be .126

Apr 4/07 Page SI-51, Problem 5 solution,

$E[S_2 \mid S_2 > 80]$, should be $121 \cdot \dfrac{\Phi(1.01)}{\Phi(.45)} = 151.6$

$E[S_2 \mid S_2 > 125]$, should be $121 \cdot \dfrac{\Phi(.23)}{\Phi(-.34)} = 212.3$

Apr 4/07 Page SI-51, Problem 6 solution,

$$E[S_1 \mid S_1 < 80] \text{ , should be } 110 \cdot \frac{\Phi\left(\dfrac{\ln 80 - \ln 100 - (\ln 1.1 + \frac{1}{2}(.16))}{.4}\right)}{\Phi\left(\dfrac{\ln 80 - \ln 100 - (\ln 1.1 - \frac{1}{2}(.16))}{.4}\right)} = 110 \cdot \frac{\Phi(-1.00)}{\Phi(-.60)} = 63.6$$

and $E[S_2 \mid S_1 < 80] = 1.1(63.6) = 70.0$.

Also

$$E[S_1 \mid S_1 > 125] \text{ , should be } 110 \cdot \frac{\Phi\left(\dfrac{\ln 100 - \ln 125 + (\ln 1.1 + \frac{1}{2}(.16))}{.4}\right)}{\Phi\left(\dfrac{\ln 100 - \ln 125 + (\ln 1.1 - \frac{1}{2}(.16))}{.4}\right)} = 110 \cdot \frac{\Phi(-.12)}{\Phi(-.52)} = 165.0$$

and $E[S_2 \mid S_1 > 125] = 1.1(165.0) = 181.5$

May 12/07 Page SI-51, Problem 8 solution, answer should be 8.3 (not 5.6)

May 12/07 Page SI-52, Problem 10 solution, in line 1 , 1/16 should be 6.25%
In line 3, .434 should be .489 and .157 should be .156

May 12/07 Page SI-59, Example SI-15 solution, in line 3 , .9083 should be .9531,
in line 5, .20 should be .222
and in line 8, .2 in the exponent should be .222 and the answer should be 112.93

May 12/07 Page SI-62, Problem 4 solution should be as follows, last line, answer should be 111.66
The model for the stock price at time 2 is

$$\hat{S}_2 = S_0 e^{(\alpha - \lambda k - \frac{1}{2}\sigma^2)(2) + \sigma\sqrt{2}Z} \cdot e^{m(\alpha_J - \frac{1}{2}\sigma_J^2) + \sigma_J \sum_{i=1}^{m} W_i} .$$

For this model, $\lambda = 1$ is the average number of jumps per year.
m has a Poisson distribution with a mean of $\lambda h = 1 \times 2 = 2$ jumps in 2 years.
$k = e^{\alpha_J} - 1 = e^{.05} - 1 = .051271$ and $e^{\alpha} = 1.1$.
We are also given $\sigma = .4$, $\sigma_J = .2$ and $\delta = 0$. Substituting in these values results in

$$\hat{S}_2 = 100(1.1)^2 e^{(-.0513 - \frac{1}{2}(.16))(2) + .4\sqrt{2}Z} \cdot e^{2(.05 - \frac{1}{2}(.04)) + .2\sum_{i=1}^{2} W_i} \quad \text{(since } e^{\alpha} = 1.1\text{)}.$$

The uniform value .61 simulates a standard normal Z of .279.
The cdf of the Poisson distribution with mean 2 is
$F(0) = .1353, F(1) = .4060, F(2) = .6767$, so the uniform number .65 simulates $m = 2$ jumps,
so we need two standard normal W_i 's. The uniform value .89 simulates $W_1 = 1.23$ and .17 simulates $W_2 = -.954$. The simulated stock price is

$$121 e^{(-.0513 - \frac{1}{2}(.16))(2) + .4\sqrt{2}(.279)} \cdot e^{2(.05 - \frac{1}{2}(.04)) + .2(1.23 - .954)} = 122.27 .$$

Mar 8/07 Page SI-66, Example SI-16,
(a) θ should be 1000 (not 2000)
Solution to (c), the answer should b 3302.6 (not .3302.6)
Solution to (d), line 2, 4639 should be 8374

Apr 4/07 Page SI-68, Dual Power Transform, 3rd last line
$1 - [F_L(t)]^{1/\kappa}$ should be $1 - [F_L(t)]^{\kappa}$

Mar 13/07 Page SI-72, #1 solution,
(b) in lines 4 and 5, 182,079 should be 57,721.7, and 228,495 should be 79,266
(c) in line 4, .223 should be $\sqrt{.223}$ and in line 5, 9.641 should be 10.197
and 15,383 should be 26,833

Mar 13/07 Page SI-72, #2 solution,
3^{rd} line from bottom page, 946/729 should be 512/729
2^{nd} line from bottom of page, 946 should be 512 (two places) and 14.78 should be 8
last line, 946/64 should be 8 and 13.6 should be 7.6

Jan 16/07 Page PE-179, #19, fourth line should be
(ii) Observations other than 0 and 1 have been deleted from the data.

May 8/07 Page PE-216, #8, Answer E should be 768

May 8/07 Page PE-225, #8 solution, last line should be
$= 18 + 25(6) + 100(6) = 768$. Answer: E