EXAM M QUESTIONS OF THE WEEK

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Week of February 13/06

A portfolio of life insurance policies consists of policies written on both smokers and non-smokers. Smokers make up 1/3 of the policies. Half of the policies written on smokers have a face amount of $100,000 and half have a face amount of $200,000. One-quarter of the policies written on non-smokers have a face amount of $100,000 and three-quarters have a face amount of $200,000. Death benefits are paid at the moment of death. The life insurance actuary models mortality for smokers as have a constant force of mortality of .02, and mortality for non-smokers has a constant force of mortality of .01. Insurance is calculated based on a force of interest of .05. An insurance policy is chosen at random from the portfolio. Find the standard deviation of the present value random variable of the benefit paid.

The solution can be found below.

Week of February 13/06 - Solution

The portfolio of policies can be split into four groups:
smoker and face amount 100,000 - 1/6 of all policies ,
smoker and face amount 200,000 - 1/6 of all policies ,
non-smoker and face amount 100,000 - 1/6 of all policies ,
non-smoker and face amount 200,000 - 1/2 of all policies .

Let W represent the present value random variable of the benefit paid. Then W is a mixture of the four present value random variables:
U_1 , smoker, face amount 100,000, mixing weight $\frac{1}{6}$,
U_2 , smoker, face amount 200,000, mixing weight $\frac{1}{6}$,
U_3 , non-smoker, face amount 100,000 , mixing weight $\frac{1}{6}$,
U_4 , smoker, face amount 200,000 , mixing weight $\frac{1}{2}$.

The mean of W is the mixture of the means of the 4 policy types.
We use the formulation for continuous insurance for constant force of mortality,
$E[Z] = \overline{A}_x = \frac{\mu}{\mu+\delta}$.
$E[W] = \frac{1}{6} \cdot E[U_1] + \frac{1}{6} \cdot E[U_2] + \frac{1}{3} \cdot E[U_3] + \frac{1}{3} \cdot E[U_4]$
$= \frac{1}{6} \cdot 100,000 \cdot \frac{.02}{.02+.05} + \frac{1}{6} \cdot 200,000 \cdot \frac{.02}{.02+.05} + \frac{1}{6} \cdot 100,000 \cdot \frac{.01}{.01+.05} + \frac{1}{2} \cdot 200,000 \cdot \frac{.01}{.01+.05}$
$= 33,730$.

The second moment of W is the mixture of the second moments of the 4 policy types.
We use the formulation for continuous insurance for constant force of mortality,
$E[Z^2] = {}^2\overline{A}_x = \frac{\mu}{\mu+2\delta}$.
$E[W^2] = \frac{1}{6} \cdot E[U_1^2] + \frac{1}{6} \cdot E[U_2^2] + \frac{1}{3} \cdot E[U_3^2] + \frac{1}{3} \cdot E[U_4^2]$
$= \frac{1}{6} \cdot 100,000^2 \cdot \frac{.02}{.02+.10} + \frac{1}{6} \cdot 200,000^2 \cdot \frac{.02}{.02+.10} + \frac{1}{6} \cdot 100,000^2 \cdot \frac{.01}{.01+.10} + \frac{1}{2} \cdot 200,000^2 \cdot \frac{.01}{.01+.10}$
$= 3,358,585,859$.

The variance of W is $Var[W] = E[W^2] - (E[W])^2 = 2,220,862,250$,
and the standard deviation of W is $47,126$.