EXAM FM QUESTION OF THE WEEK

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Week of March 17/08

A bond has annual coupons at rate $r > .05$, matures at par and has an annual effective yield rate of 8.2%.

D_{24} is the Macaulay duration of the bond if it matures in 24 years,
D_{25} is the Macaulay duration of the bond if it matures in 25 years, and
D_{26} is the Macaulay duration of the bond if it matures in 26 years.

You are given that $D_{25} - D_{24} = .14583$.

Determine $D_{26} - D_{25}$.

The solution can be found below.

Week of March 17/08 - Solution

Suppose the face amount of the bond is 1.

Then $D_{24} = \frac{r(Ia)_{\overline{24}|.082}+24v_{.082}^{24}}{ra_{\overline{24}|.082}+v_{.082}^{24}} = \frac{92.4906r+3.62040}{10.3555r+.150850}$, and

$D_{25} = \frac{r(Ia)_{\overline{25}|.082}+25v_{.082}^{25}}{ra_{\overline{25}|.082}+v_{.082}^{25}} = \frac{95.9761r+3.48545}{10.4949r+.139418}$.

We are given that $\frac{95.9761r+3.48545}{10.4949r+.139418} - \frac{92.4906r+3.62040}{10.3555r+.150850} = .14583$.

This leads to the quadratic equation

$(95.9761r+3.48545)(10.3555r+.150850)$

$-(92.4906r+3.62040)(10.4949r+.139418)$

$= .14583(10.4949r+.139418)(10.3555r+.150850)$,

or equivalently,

$$7.3521r^2 - .7604r + .0180 = 0 .$$

Solving for r results in $r = .0367$ or $r = .0667$.

We ignore the smaller root.

Then $D_{26} = \frac{.0667(Ia)_{\overline{25}|.082}+25v_{.082}^{25}}{.0667a_{\overline{25}|.082}+v_{.082}^{25}} = 11.911$.

From $r = .0667$, we get $D_{25} = 11.778$, so that $D_{26} - D_{25} = .13$.