EXAM FM QUESTIONS OF THE WEEK

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Week of March 5/07

An actuary is analyzing price and duration of three separate bonds, Bond A, Bond B and Bond C, all based on a flat term structure (same yield to maturity for all bonds). Bond B has a price of 110 and a Macaulay duration of 11 years and Bond C has a price of 95 and a Macaulay duration of 14 years. The portfolio resulting from the combination of Bond A and Bond B has a Macaulay duration of 11.49 years, and the portfolio resulting from the combination of all three bonds has a Macaulay duration of 12.26 years. Find the price and Macaulay duration of Bond A.

The solution can be found below.

Week of March 5/07 - Solution

The duration of a set of cashflows is the "weighted average time to maturity" of the set of cashflows. Larger payments at the start of the series of cashflows tends to make the duration lower and larger payments near the end of the series tends to make the duration larger. We note that the three series of cashflows all have the same total amount of 550 paid, so we would expect that the decreasing annuity has the smallest duration, the level annuity has the next largest duration and the increasing annuity would have the largest duration, $D_{III} < D_I < D_{II}$.
The actual numerical of the Macaulay durations are

$$D_I = \frac{v+2v^2+3v^3+\cdots+10v^{10}}{v+v^2+v^3+\cdots+v^{10}} = \frac{(Ia)_{\overline{n}|}}{a_{\overline{n}|}} = 4.73$$

$$D_{II} = \frac{v+4v^2+9v^3+\cdots+10v^{10}}{v+2v^2+3v^3+\cdots+10v^{10}} = \frac{v+4v^2+9v^3+\cdots+10v^{10}}{(Ia)_{\overline{n}|}} = 6.39$$

$$D_{III} = \frac{10v+18v^2+24v^3+\cdots+10v^{10}}{10v+9v^2+8v^3+\cdots+v^{10}} = \frac{10v+18v^2+24v^3+\cdots+10v^{10}}{(Da)_{\overline{n}|}} = 3.47$$

The numerators of D_{II} and D_{III} can be found by direct calculation.