EXAM P QUESTIONS OF THE WEEK

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Week of January 23/06

X and Y have a joint distribution on the two-dimensional region $-1 \leq x \leq 1$, $-2 \leq y \leq 2$, and the pdf of the joint distribution is $f(x,y) = c$ (constant) on the region.
Find the probability $P(|X| \leq |Y|)$.

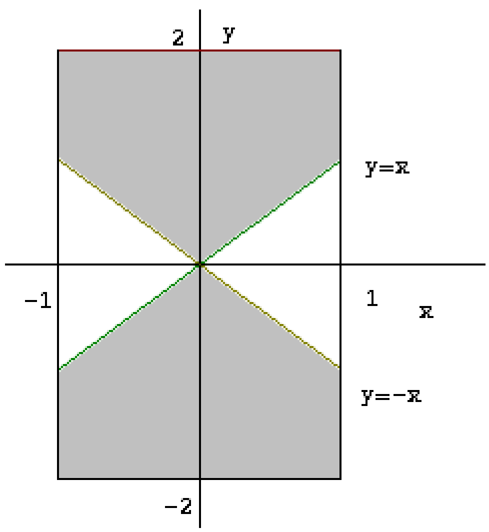
The solution can be found below.

<u>Week of January 23/06 - Solution</u>

Whenever a joint distribution has a constant density over the entire probability space S, the probability of any subregion A is equal to $\frac{\text{Area of } A}{\text{Area of } S}$.

The area of the full probability space is $2 \times 4 = 8$ (the area of a rectangle with sides that are 2.

The graph of the region defined by $|X| \leq |Y|$ is the shaded region illustrated below
To find this region, we first find the boundary $|X| = |Y|$, which is the combination of the two lines $y = x$ and $y = -x$. We then determine "which sides" of the lines represent the inequality. Alternatively, we consider the relationship between x and y in the four quadrants:
(i) 1st quadrant, $x \geq 0$, $y \geq 0$ so $|x| = x$, $|y| = y$ and the inequality becomes $x \leq y$,
(ii) 2nd quadrant, $x \leq 0$, $y \geq 0$ so $|x| = -x$, $|y| = y$, the inequality becomes $-x \leq y$,
(iii) 3rd quadrant, $|x| = -x$, $|y| = -y$, the inequality becomes $-x \leq -y$ (or $x \geq y$), and
(iv) 4th quadrant, $|x| = x$, $|y| = -y$, the inequality becomes $x \leq -y$ (or $x \geq y$) .



The area of the shaded region is $8 -$ (Area of blank region) , and the area of the blank region is 2 (two triangles, each with base 2 and height 1), so the area of the shaded region is 6.
The probability is $P(|X| \leq |Y|) = \frac{6}{8}$.

Note that since the joint pdf is the constant c, it must be true that
$c = \frac{1}{\text{Area of Total Region of Probability}} = \frac{1}{2 \times 4} = \frac{1}{8}$.
The probability would be the double integral of the joint pdf over the shaded region.
Since the joint pdf is constant at $\frac{1}{8}$, the integral over the shaded region would be
$\frac{1}{8} \times$ (Area of shaded region) .