

EXAM C QUESTIONS OF THE WEEK

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Week of January 2/06

A ground up loss distribution is assumed to follow a Pareto distribution with $\alpha = 2$, and unknown θ . A set of 10 observations based on policy limit amount L has been used to estimate θ using moment estimation. There are 6 observations below the limit L and they total 2350. There are 4 limit observations of limit amount L . L is known to be less than 1000. The moment estimate of θ is $\hat{\theta} = 1866.3$. Using the estimated Pareto distribution, find the estimate of the probability that a ground up loss is greater than the limit amount L .

Solution can be found below.

Week of January 2/06 - Solution

We are given that $\hat{\theta} = 1866.3$.

We wish to find the probability $P(X > L) = \left(\frac{\theta}{L+\theta}\right)^2$.

If we knew the value of L , then we could find this probability.

With $\alpha = 2$, the limited expected value for a Pareto distribution with limit L is

$$\theta\left[1 - \frac{\theta}{L+\theta}\right] = \frac{L\theta}{L+\theta}.$$

According to the method of moments applied for limited expected value, we set the limited expected value of the Pareto distribution equal to the empirical limited expected value:

$$\frac{L\theta}{L+\theta} = \frac{x_1 + \dots + x_6 + 4L}{10} = \frac{2350 + 4L}{10}. \text{ Using the value of } \hat{\theta} = 1866.3, \text{ this equation becomes}$$
$$4L^2 - 8847.8L + 4,385,805 = 0.$$

Solving this quadratic equation results in two values of L : 750 and 1462.

We are told that the limit is below 1000, so we choose $L = 750$.

The estimate of $P(X > L)$ is $\left(\frac{1866.3}{750+1866.3}\right)^2 = .51$.