EXAM FM QUESTIONS OF THE WEEK

Week of January 30/06

Smith borrows $10,000 at annual effective rate of interest 8%. He will pay interest to the lender at the end of each year, and will make principal payments of $3,000 at the end of 5 years and $7,000 at the end of 10 years. He considers two different ways of contributing to a sinking fund that earns an annual effective rate of interest of 5% that will be accumulated to pay the principal payments when they are due:

Alternative 1:
He separates the original loan into two loans, one of $3,000, and one of $7,000. He makes a level sinking fund deposit at the end of each year of X for 5 years to accumulate to pay the loan of $3,000 due in 5 years, and he makes another level sinking fund deposit of Y at the end of each year for 10 years to accumulate to pay the loan of $7,000 due in 10 years

Alternative 2:
He makes a level deposit of Z at the end of each year for 10 years. At the end of 5 years, he withdraws $3,000 from the sinking fund to pay the loan of $3,000 due then. The accumulated amount in the account at the end of 10 years is just enough to pay the $7,000 due then.

A_1 and A_2 denote Smith's total payments (interest plus sinking fund deposits) under Alternatives 1 and 2 respectively. Find A_1/A_2 .

The solution can be found below.

Week of January 30/06 - Solution

Under both alternatives, Smith's interest payments are $10,000 \times .08 = 800$ per year for the first 5 years, and they are $7,000 \times .08 = 560$ per year for the final 5 years.

Under Alternative 1 Smith's sinking fund deposits are $X = \frac{3000}{s_{\overline{5}|.05}} = 542.92$, paid each year for 5 years, and $Y = \frac{7000}{s_{\overline{10}|.05}} = 556.53$ per year for 10 years .

Smith's total payments under Alternative 1 are

$800 \times 5 + 560 \times 5 + 542.92 \times 5 + 556.53 \times 10 = 15,079.90$

Under Alternative 2, Smith's sinking fund deposits must satisfy the relationship

$Y s_{\overline{10}|.05} - 3000(1.05)^5 = 7000$, since there is a withdrawal of 3000 at the end of 5 years.

Solving for Y results in $Y = 860.94$.

Smith's total payments under Alternative 2 are

$800 \times 5 + 560 \times 5 + 860.94 \times 10 = 15,409.40$.

$A_1/A_2 = .9786$.