EXAM C QUESTIONS OF THE WEEK

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Week of January 9/06

The method of percentile matching is applied to estimate the parameter p for the random variable with pdf $f(x) = (p+1)x^p$, $0 < x < 1$. Percentile matching is done at the median, using the smoothed empirical estimate of the sample median from a random sample. Based on the estimated value of p, it is found that the mean of the random variable is .75647 . Find the smoothed empirical estimate of the sample median.

Solution can be found below.

Week of January 9/06 - Solution

We denote the smoothed empirical estimate of the sample median by m.

The cdf of X is $F(t) = \int_0^t (p+1)x^p\,dx = t^{p+1}$.

According to the method of percentile matching, we substitute the sample median m into the distribution function, and set it equal to .5, $m^{p+1} = .5$.

From this, we get $p = \frac{\ln .5}{\ln m} - 1$.

The mean of X is $\int_0^1 x \cdot (p+1)x^p\,dx = \frac{p+1}{p+2}$.

We are given that the estimated mean is .75647, so that $\frac{p+1}{p+2} = .75647$.

Solving for p results in $p = 2.1063$.

Then from $p = \frac{\ln .5}{\ln m} - 1$, we get $m = .80$.