EXAM FM QUESTIONS OF THE WEEK

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Week of July 10/06

Smith purchases an inflation indexed annuity that will make payments
at the end of each year for 20 years. The first payment, due 10 years from now
will be $50,000. For the following 9 years each payment will be 6% larger
than the previous payment. For the 10 years after that, each payment will be
3% larger than the previous payment. The annuity is valued using the following
annual effective rates of interest, with time measured from now:
8% per year for the next 20 years, 5% per year for the 10 years after that.

Find the present value of the annuity now.

The solution can be found below.

Week of July 10/06 - Solution

The first 10 payments are
$$50,000 \ , \ 50,000(1.06) \ , \ 50,000(1.06)^2 \ , \ \ldots \ , \ 50,000(1.06)^9$$
made at times 10 , 11 , 12 , . . . , 19 .

The second 10 payments are
$$50,000(1.06)^9(1.03) \ , \ 50,000(1.06)^9(1.03)^2 \ , \ \ldots, \ 50,000(1.06)^9(1.03)^{10}$$
made at times 20 , 21 , . . . , 29 .

The value at time 10 of the first 10 payments is
$$50,000(1.08) \cdot \frac{1-\left(\frac{1.06}{1.08}\right)^{10}}{.08-.06} = 460,325.72 \ ,$$
so that value at time 0 of the first 10 payments is $460,325.72 v_{.08}^{10} = 213,220$.

The value at time 20 of the second 10 payments is
$$50,000(1.06)^9(1.03)(1.05) \cdot \frac{1-\left(\frac{1.03}{1.05}\right)^{10}}{.05-.03} = 799,167.92 \ ,$$
so the value at time 0 of the second 10 payments is $799,167.92 v_{.08}^{20} = 171,460$.

Total present value at time 0 is $213,220 + 171,460 = 384,680$.