EXAM P QUESTIONS OF THE WEEK

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Week of July 10/06

X has a discrete non-negative integer valued distribution with a mean of 5 and a variance of 10.

Two new distributions are created from X.

Y has the same probability function as X for $Y = 2, 3, 4, \dots$, but $P(Y = 0) = 0$ and
$P(Y = 1) = P(X = 0) + P(X = 1)$.

Z has the same probability function as X for $Z = 2, 3, 4, \dots$, but
$P(Z = 0) = P(Z = 1) = 0$ and $P(Z = 2) = P(X = 0) + P(X = 1) + P(X = 2)$

You are given that the mean of Y is 5.1 and the mean of Z is 5.3 .

Find the variance of Z.

The solution can be found below.

Week of July 10/06 - Solution

We wish to find $Var[Z] = E[Z^2] - (E[Z])^2 = E[Z^2] - (5.3)^2$.

Let us denote $P(X = 0) = p_0$, $P(X = 1) = p_1$, $P(X = 2) = p_2$, etc.

Then $P(Y = 0) = 0$, $P(Y = 1) = p_0 + p_1$, $P(Y = 2) = p_2$, etc.

and

Then $P(Z = 0) = P(Z = 1) = 0$, $P(Z = 2) = p_0 + p_1 + p_2$,
$P(Y = 3) = p_3$, etc.

$5.0 = E[X] = p_1 + 2p_2 + 3p_3 + \cdots$ and
$5.1 = E[Y] = (p_0 + p_1) + 2p_2 + 3p_3 + \cdots$ and
$5.3 = E[Z] = 2(p_0 + p_1 + p_2) + 3p_3 + \cdots$.

Therefore, $.1 = E[Y] - E[X] = p_0$ and $.2 = E[Z] - E[Y] = p_0 + p_1 = .1 + p_1$,
so that $p_1 = .1$.

From $10 = Var[X] = E[X^2] - (E[X])^2 = E[X^2] - 25$, we have
$E[X^2] = 35 = p_1 + 4p_2 + 9p_3 + \cdots$
Then, $E[Z^2] = 4(p_0 + p_1 + p_2) + 9p_3 + \cdots = 4p_0 + 3p_1 + E[X^2]$
$\quad = 4(.1) + 3(.1) + 35 = 35.7$, and
$Var[Z] = 35.7 - (5.3)^2 = 7.61$.

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