EXAM C QUESTIONS OF THE WEEK

S. Broverman, 2006

Week of July 17/06

A mortality study estimates the survival distribution of a rat that has just been diagnosed with mad rat disease. The study is based on 80 times of death summarized in monthly grouped form. There is no censoring or truncation in the study. The data summary is:

Month	1	2	3	4	5	6	7	8
# of deaths	15	5	10	20	10	10	5	5

Find the log-transformed 95% confidence interval for $S(2.5)$.

Solution can be found below.

Week of July 17/06 - Solution

The interval has lower and upper limits of $[S_{80}(2.5)]^{1/U}$ and $[S_{80}(2.5)]^{U}$, respectively,

where $U = exp\left[\frac{1.96\sqrt{V\widehat{a}r[S_{80}(2.5)]}}{S_{80}(2.5)\cdot ln[S_{80}(2.5)]}\right]$.

The first month corresponds to the interval $(0, 1]$, the second month $(1, 2]$, etc.

Using the grouped data method, we have $S_{80}(2.5) = \frac{3-2.5}{3-2} \cdot S_{80}(2) + \frac{2.5-2}{3-2} \cdot S_{80}(3)$.

Since there are 60 survivors to the end of the interval $(1, 2]$ and 50 survivors to the end of interval $(2, 3]$, we have $S_{80}(2) = \frac{60}{80}$ and $S_{80}(3) = \frac{50}{80}$, so that $S_{80}(2.5) = .6875 = \frac{11}{16}$.

The estimated variance of $S_{80}(2.5)$ is

$V\widehat{a}r[S_{80}(2.5)] = \frac{(3-2)^2 V\widehat{a}r[Y] + (2.5-2)^2 V\widehat{a}r[Z] + 2(3-2)(2.5-2)C\widehat{o}v[Y,Z]}{[80(3-2)]^2}$

$= \frac{1}{80^2} \cdot (V\widehat{a}r[Y] + .25V\widehat{a}r[Z] + C\widehat{o}v[Y,Z])$,

where $V\widehat{a}r[Y] = 80 \cdot S_{80}(2) \cdot [1 - S_{80}(3)] = 15$,

$V\widehat{a}r[Z] = 80 \cdot [S_{80}(2) - S_{80}(3)] \cdot [1 - S_{80}(2) + S_{80}(3)] = 8.75$, and

$C\widehat{o}v[Y, Z] = -80 \cdot [1 - S_{80}(2)] \cdot [S_{80}(2) - S_{80}(3)] = -2.5$.

Then, $V\widehat{a}r[S_{80}(2.5)] = .002295$.

In the confidence interval calculations, we get $U = .6945$.

The lower limit of the interval is $(\frac{11}{16})^{1/.6945} = .583$,

and the upper limit is $(\frac{11}{16})^{.6945} = .771$.