

EXAM FM QUESTIONS OF THE WEEK

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Week of July 17/06

Smith deposits \$100,000 into a bank account that pays 1% interest at the end of every month. At the end of each month, Smith withdraws the interest payment for that month and deposits into an account earning 2% per month. At the end of each 12th month, along with the withdrawal of interest, Smith withdraws \$10,000 from the account. The \$10,000 withdrawal is deposited into an account earning an effective annual interest rate of 10%. Find Smith's annual effective yield for the 10 year period.

The solution can be found below.

Week of July 17/06 - Solution

The accumulated value of the reinvested annual \$10,000 withdrawals is

$$10,000s_{\overline{10}|.10} = 159,374.25 .$$

The interest in the first year is $100,000 \times (.01) = 1000$ per month, so at the end of the first year, the reinvested interest has accumulated to $1000s_{\overline{12}|.02} = 13,412.09$ as of the end of the first year.

The interest in the second year is $90,000 \times (.01) = 900$ per month, so at the end of the first year, the reinvested interest has accumulated to $900s_{\overline{12}|.01} = (13,412.09)(.9) = 12,070.88$ as of the end of the second year.

In a similar way, the interest from the 3rd year accumulates to $(13,412.09)(.8) = 10,729.67$ at the end of the 3rd year, . . . , and the interest in the 10 year accumulates to $(13,412.09)(.1) = 1,341.21$ at the end of the 10th year.

The reinvested interest forms a 10-year decreasing annuity-immediate with payments $1,341.21 \times 10, 1,341.21 \times 9, \dots, 1,341.21 \times 1$., and the deposits are accumulating at 2% per month, or 26.824% as an effective annual rate. The reinvested interest accumulates to $1,341.21 \cdot (Ds)_{\overline{10}|.26824} = 356,233$.

The total accumulated value at the end of 10 years is $159,374 + 356,233 = 515,607$.

Smith's annual effective yield is j , where $100,000(1 + j)^{10} = 515,607$.

Solving for j results in $j = .178$.

I thank Mr. H. Forman for providing an alternative approach (he also pointed out an error in the solution when the problem was first posted). Mr. Forman has calculated the accumulated interest in the following way:

$$1000s_{\overline{120}|.02} - 100[s_{\overline{108}|.02} + s_{\overline{96}|.02} + s_{\overline{84}|.02} + s_{\overline{72}|.02} + s_{\overline{60}|.02} + s_{\overline{48}|.02} + s_{\overline{36}|.02} + s_{\overline{24}|.02} + s_{\overline{12}|.02}]$$

The net pattern of interest reinvestment is 1000 per month in the first year, followed by $1000 - 100 = 900$ per month in the second year, etc.