EXAM M QUESTIONS OF THE WEEK

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Week of July 17/06

A fully discrete 20-year deferred life annuity-due is issued to (40).
The annuity will pay \$100,000 every year starting at age 60.
The annuity will have level annual benefit premiums payable for 10 years starting at age 40.
There is no benefit during the deferral period.

Mortality is based on the Illustrative Table (download from the SOA website at the Exam M webpage) with effective annual interest of 6%

Find the maximum age (y) so that of death occurs after age (y) but before age $(y+1)$, the issue date loss is negative.

The solution can be found below.

Week of July 17/06 - Solution

The annual premium is P, which is the solution of the equation

$P \cdot \ddot{a}_{40:\overline{10}|} = 100,000 \cdot {}_{20|}\ddot{a}_{40}$, which can be written as

$P \cdot (\ddot{a}_{40} - {}_{10}E_{40} \cdot \ddot{a}_{50}) = 100,000 \cdot {}_{20}E_{40} \cdot \ddot{a}_{60}$.

Using values from the Illustrative Table, solving for P results in $P = 39,687.50$.

The issue date loss is PV of benefit $-$ PV of premium .
If death occurs before age 60, there is no benefit, so the loss is $0 -$ PV of premium < 0 .

If death occurs between age $60 + n$ and $60 + n + 1$, there will be $n + 1$ annuity payments.
The PV at age 40 of the annuity payments will be $100,000v^{20} \cdot \ddot{a}_{\overline{n+1}|.06} = 31,180.47 \cdot \ddot{a}_{\overline{n+1}|.06}$.
The present value at age 40 of the premiums (if all 10 premiums are paid) is
$39,687.50 \cdot \ddot{a}_{\overline{10}|.06} = 309,629.66$.

If death occurs between age $60 + n$ and $60 + n + 1$, the issue date loss is
$31,180.47 \cdot \ddot{a}_{\overline{n+1}|.06} - 309,629.66$.
This will be negative if $\ddot{a}_{\overline{n+1}|.06} < 9.93$.
By trial and error, or by solving for n from $\frac{1-v^{n+1}}{d} = 9.93$,
we see that $\ddot{a}_{\overline{14}|} = 9.85$ and $\ddot{a}_{\overline{15}|} = 10.29$.
Therefore, if death occurs between ages 73 and 74, there will be 14 annuity payments, and the issue date loss will be $100,000v^{20} \cdot \ddot{a}_{\overline{14}|.06} - 309,629.66 = -2,418$,
but if death occurs after age 74, there will be at least 15 annuity payments, and the issue date loss will be at least $100,000v^{20} \cdot \ddot{a}_{\overline{15}|.06} - 309,629.66 = 11,328$.

The age y is 73.