

## EXAM P QUESTIONS OF THE WEEK

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### Week of July 17/06

The time until failure of a machine is modeled as an exponential distribution with a mean of 3 years. A warranty on the machine provides the following schedule of refunds:

- if the machine fails within 1 year, the full purchase price is refunded,
- if the machine fails after 1 year but before 2 years,  $\frac{3}{4}$  of the purchase price is refunded,
- if the machine fails after 2 years but before 4 years,  $\frac{1}{2}$  of the purchase price is refunded, and
- if the machine fails after 4 years,  $\frac{1}{4}$  of the purchase price is refunded.

Find the expected fraction of the purchase price that will be refunded under the warranty.

**The solution can be found below.**

## Week of July 17/06 - Solution

Suppose that  $T$  is the time until failure of the machine.

$$P(a < T \leq b) = e^{-a/3} - e^{-b/3} .$$

The fraction of the purchase price refunded is a random variable  $X$  that can be described in the following way:

$$X = \begin{cases} 1 & 0 < T \leq 1 & \text{prob. } 1 - e^{-1/3} \\ 3/4 & 1 < T \leq 2 & \text{prob. } e^{-1/3} - e^{-2/3} \\ 1/2 & 2 < T \leq 3 & \text{prob. } e^{-2/3} - e^{-4/3} \\ 1/4 & T > 3 & \text{prob. } e^{-4/3} \end{cases}$$

$$\begin{aligned} \text{Then, } E[X] &= 1 - e^{-1/3} + \frac{3}{4}(e^{-1/3} - e^{-2/3}) + \frac{1}{2}(e^{-2/3} - e^{-4/3}) + \frac{1}{4}e^{-4/3} \\ &= 1 - \frac{1}{4}e^{-1/3} - \frac{1}{4}e^{-2/3} - \frac{1}{4}e^{-4/3} = .627 . \end{aligned}$$