EXAM C QUESTIONS OF THE WEEK

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Week of July 24/06

X is a mixture of 3 exponential random variables, with means 1, 2 and 3.

A random sample of observed values of X has sample mean 2.4 and sample median 1.5.

A combination of the method of moments and the method percentile matching is applied to estimate the mixing weights. Find the variance of the estimated distribution.

Solution can be found below.

Week of July 24/06 - Solution

$f(x) = a_1 e^{-x} + a_2 \cdot \frac{1}{2} e^{-x/2} + (1 - a_1 - a_2) \cdot \frac{1}{3} e^{-x/3}$.
$F(x) = a_1(1 - e^{-x}) + a_2(1 - e^{-x/2}) + (1 - a_1 - a_2)(1 - e^{-x/3})$.

$E[X] = a_1 + 2a_2 + 3(1 - a_1 - a_2) = 3 - 2a_1 - a_2 = 2.4$
$\rightarrow 2a_1 + a_2 = .6$.

$F(1.5) = a_1(1 - e^{-1.5}) + a_2(1 - e^{-1.5/2}) + (1 - a_1 - a_2)(1 - e^{-1.5/3}) = .5$
$\rightarrow$ $.383401a_1 + .134164a_2 = .106531$

Solving for a_1 and a_2 results in $a_1 = .2262$, $a_2 = .1476$, $1 - a_1 - a_2 = .6262$.

The second moment of X is
$E[X^2] = a_1 \cdot 2 + a_2 \cdot 8 + (1 - a_1 - a_2) \cdot 18 = 12.9$,
and the variance of X is $12.9 - (2.4)^2 = 7.14$.