EXAM FM QUESTIONS OF THE WEEK

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Week of July 24/06

A $10,000 loan has monthly payments at the end of every month. The loan interest rate is 1% per month. At the end of each month, the borrower pays $100 plus half of the interest due for that month. Find the balance still owing just after the 12th payment.

The solution can be found below.

Week of July 24/06 - Solution

The initial loan amount is 10,000 .

The interest due at the end of the first month is $10,000(.01)$, and the payment made is $10,000(.005) + 100$. The outstanding balance just after the first monthly payment is $10,000(1.01) - 10,000(1.005) - 100 = 10,000(1.005) - 100$.

We now consider the pattern for outstanding balance from one month to the next.
Suppose that the balance just after the t-th payment is OB_t .
The interest due at time $t + 1$ is $OB_t \cdot i$, so the amount paid is $OB_t \cdot \frac{i}{2} + 100$.
The outstanding balance just after the payment at time $t + 1$ is
$OB_{t+1} = OB_t(1 + i) - OB_t \cdot \frac{i}{2} - 100 = OB_t(1 + \frac{i}{2}) - 100$.
This is the amortization pattern of a loan at interest rate $\frac{i}{2}$ and level payments of 100.

The outstanding balance just after the 12th payment will be
$10,000(1.005)^{12} - 100s_{\overline{12}|.005} = 9,383.22$.