EXAM P QUESTIONS OF THE WEEK

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Week of July 24/06

At the start of a year, Smith is presented with an investment proposal. Smith's payoff from the investment is related to the closing value of an international financial stock index on the last day of the year. If the closing value of the index on the last day of the year is X, Smith's payoff will be $Y = Min\{ \, Max\{X, 20\} \, , \, 50 \, \}$.

At the start of the year, when Smith is considering this proposal, Smith's model for X is that X has a continuous uniform distribution on the interval $(0, 100)$.

Based on Smith's model, find the expected payoff.

The solution can be found below.

Week of July 24/06 - Solution

If the index closes below 20, then $Y = Max\{X, 20\} = 20$,

and if the index closes above 50, then $Y = Min\{ Max\{X, 20\}, 50\} = 50$.

If the index closes between 20 and 50, then

$Y = Min\{ Max\{X, 20\}, 50\} = Min\{X, 50\} = X$.

Therefore, $Y = \begin{cases} 20 & X \le 20 \\ X & 20 < X \le 50 \\ 50 & X > 50 \end{cases}$.

$E(Y) = \int_0^{20} 20 \cdot f_X(x)\,dx + \int_{20}^{50} x \cdot f_X(x)\,dx + \int_{50}^{100} 50 \cdot f_X(x)\,dx$.

X has pdf $f_X(x) = \frac{1}{100} = .01$, so

$E(Y) = \int_0^{20} 20(.01)\,dx + \int_{20}^{50} x \cdot (.01)\,dx + \int_{50}^{100} 50(.01)\,dx = 4 + 10.5 + 25 = 39.5$.