EXAM FM QUESTIONS OF THE WEEK

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Question 1 - Week of July 25

Smith is offered and investment that has the following force of interest at time t: $\delta_t = \frac{.1}{1+.1t}$. The investment allows amounts to be invested at any time starting from time 0 and on. Smith decides to invest \$1,000 at time 0 and \$1,000 at time $\frac{1}{2}$, with the intention of ending the investment at time 1. Smith consults his Exam FM reference book and notices that the given force of interest corresponds to simple interest growth at an annual rate of 10% for an investment made at time 0. Smith does a quick calculation and concludes that at time 1 she will have \$2,150. Is she correct, and if not, what amount will she have when the investment ends at time 1?

The solution can be found below.

Question 1 - Week of July 25 - Solution

At the stated force of interest, the investment growth factor from time n_0 to n_1 is $e^{\int_{n_0}^{n_1} \frac{1}{1+.1t} dt} = \frac{1+.1n_1}{1+.1n_0}$. If $n_0 = 0$ then this factor is $1 + .1n_1$, which is usual simple interest growth at 10%. Smith's investment of \$1,000 made at time 0 does grow to \$1,000(1+.1) = \$1,100. However, Smith's investment of \$1,000 made at time $\frac{1}{2}$ grows to $\$1,000 \cdot \frac{1+.1}{1+.1(\frac{1}{2})} = \$1,047.62$ at time 1. Smith; s total investment value at time 1 is \$2,147.62. With the stated force of interest, simple interest growth occurs only for amounts invested at time 0.