EXAM P QUESTIONS OF THE WEEK

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Question 1 - Week of July 25

A survey of the public determines the following about the "Lord of the Rings" trilogy (3 movies).

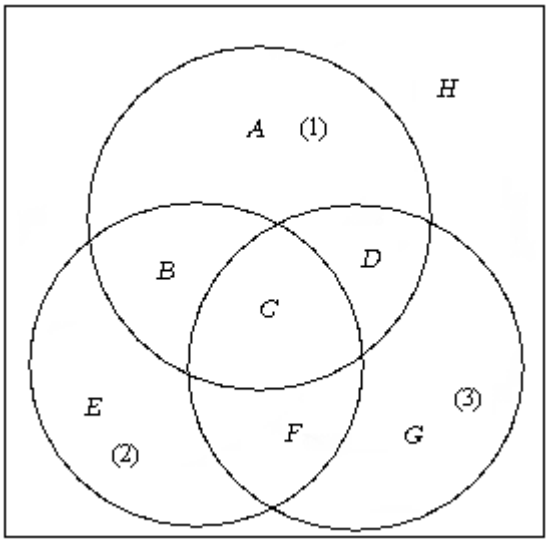
Have Seen #1	Have Seen #2	Have Seen #3	Percentage of Public
No	No	No	50%
Yes	?	?	35%
?	Yes	?	33%
?	?	Yes	31%
Yes	No	No	8%
Yes	Yes	No	4%
Yes	Yes	Yes	20%

Based on this information, determine the percentage of the public that has seen exactly one of the three "Lord of the Rings" movies.

The solution can be found below.

Question 1 - Week of July 25 - Solution

We can represent the events in the following diagram:



The top circle, $A \cup B \cup C \cup D$ represents the event of having seen #1 of the movie series, the lower left circle, $E \cup B \cup C \cup F$ represents the event of having seen #2 of the movie series, the lower right circle, $G \cup F \cup C \cup D$ represents the event of having seen #3 of the movie series, and H represents the event of having seen none of the three movies.

From the given information, we know that the percentage for event H is $h = 50$.
The second line of the information table indicates that 35% of the public has seen movie #1 but we don't know about movies #2 and #3 for this group. This is interpreted as the percentage for $A \cup B \cup C \cup D$ is $a + b + c + d = 35$.

Similarly, the percentage for $E \cup B \cup C \cup F$ is $e + b + c + f = 33$, and
the percentage for $G \cup F \cup C \cup D$ is $g + f + c + d = 31$.
The 5th line of the table indicates that 8% have seen movie #1 and not movies #2 or #3.
Therefore, the percentage for event A is $a = 8$.
Event B is the event of having seen both #1 and #2 but not #3 and this has percentage $b = 4$,
and event C is the event of have seen all three, and this has percentage $c = 20$.
The event of having seen exactly one of the three movies is the combination $A \cup E \cup G$.
This will be $a + e + g$.

We know that $a+b+c+d+e+f+g+h=100$ percent, since everyone either sees a movie or doesn't. This leads to the following 8 equations:
$h=50$ (1) , $a+b+c+d=35$ (2) , $e+b+c+f=33$ (3) , $g+f+c+d=31$ (4) , $a=8$ (5) , $b=4$ (6) , $c=20$ (7) , $a+b+c+d+e+f+g+h=1$ (8) .
From equations (3), (6) and (7) we get $e+f=9$ (9) .
From equations (1), (2) and (8) we get $e+f+g=15$ (10) .
From equations (9) and (10) we get $g=6$ (11) .
From equations (2), (5) and (6) we get $c+d=23$ (12) .
From equations (11), (12) and (4) we get $f=2$ (13) .
From equations (9) and (13) we get $e=7$.

Then $a+e+g=8+7+6=21$ is the percentage that has seen exactly one of the three movies.

Once we have determined the individual values of a,b,c,d,e,f,g,h , we can find the percentage for any combination. For instance, the percentage of people who have seen #1 and #3 but not #2 is $d=3$. Answer: D