EXAM C QUESTIONS OF THE WEEK

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Week of July 31/06

The following six losses were observed in 2005:

100 , 300 , 300 , 400 , 700 , 1000 .

The following three losses have been observed in 2006:

200 , 300 , 800 .

The model for the loss random variable in 2005 is exponential with a mean of θ.
It is assumed that there is 10% inflation in loss amounts from 2005 to 2006.

Two actuaries estimate θ using maximum likelihood estimation.

Actuary A uses an exponential distribution with mean 1.1θ as the model for the three losses in 2006 combined with the data for 2005 to estimate θ. The estimate is $\widehat{\theta}_A$.

Actuary B also uses an exponential distribution with mean 1.1θ as the model for losses in 2006. Actuary B scales each of the losses for 2005 by 10% and uses those as equivalent losses in 2006 along with the actual three losses in 2006 to estimate θ. The estimate is $\widehat{\theta}_B$.

Find $\widehat{\theta}_B - \widehat{\theta}_A$.

Solution can be found below.

Week of July 31/06 - Solution

The pdf of the 2005 loss random variable is $\frac{1}{\theta}e^{x/\theta}$, and the log is $-\,ln\,\theta\,-\,\frac{x}{\theta}$.
The pdf of the 2006 loss random variable is $\frac{1}{1.1\theta}e^{x/1.1\theta}$, and the log is $-\,ln\,1.1\theta\,-\,\frac{x}{1.1\theta}$.

We denote losses as $x_1, ..., x_6$ in 2005 and y_1, y_2, y_3 in 2006 .

For Actuary A, the log-likelihood for the 2005 data is $-\,6\,ln\,\theta\,-\,\frac{\Sigma x_i}{\theta}$,
and for the 2006 data it is $-\,3\,ln\,1.1\theta\,-\,\frac{\Sigma y_i}{1.1\theta}$.
Actuary A's total log-likelihood is
$-\,6\,ln\,\theta\,-\,\frac{\Sigma x_i}{\theta}-3\,ln\,1.1\theta\,-\,\frac{\Sigma y_i}{1.1\theta}=\,-\,3\,ln\,1.1\,-\,9\,ln\,\theta\,-\,\frac{\Sigma x_i}{\theta}-\frac{\Sigma y_i}{1.1\theta}$.

Actuary B uses scaled losses $1.1x_1,\,\ldots,\,1.1x_6$ and losses y_1, y_2, y_3 for the exponential distribution with mean 1.1θ . The log-likelihood function for Actuary B is
$-\,9\,ln\,1.1\theta\,-\,\frac{1.1\Sigma x_i}{1.1\theta}-\frac{\Sigma y_i}{1.1\theta}=\,-\,9\,ln\,1.1\,-\,9\,ln\,\theta\,-\,\frac{\Sigma x_i}{\theta}-\frac{\Sigma y_i}{1.1\theta}$.

Since the two log-likelihood functions differ by a constant, they will both be maximized at the same value of θ. Therefore $\widehat{\theta}_A=\widehat{\theta}_B$.