EXAM FM QUESTIONS OF THE WEEK

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Week of July 3/06

A perpetuity-due has annual payments that follow the pattern A, B, A, B, ...

If the annual effective rate of interest is 10%, the present value of the perpetuity is 24,619.05 .

If nominal annual interest rate compounded semi-annually is 10%, the present value of the perpetuity is 24,070.12 .

Find the present value of the perpetuity if the nominal annual interest rate is 10% compounded quarterly.

The solution can be found below.

Week of July 3/06 - Solution

If the annual effective rate of interest is i, then the 2-year effective interest rate is $(1+i)^2 - 1 = 2i + i^2$ and the 2-year effective discount rate is $d' = \frac{2i+i^2}{1+2i+i^2} = \frac{2i+i^2}{(1+i)^2}$.

The perpetuity can be split into two separate perpetuities.
The first is a perpetuity-due paying A every 2 years.
The second is a perpetuity paying B every 2 years, with the first payment one year from now.

The present value of the first perpetuity is $\frac{A}{d'}$, where d' is the 2-year discount rate,
so the present value is $\frac{(1+i)^2}{2i+i^2} \cdot A$.
The present value of the second perpetuity is $\frac{1}{1+i} \cdot \frac{B}{d'} = \frac{1}{1+i} \cdot \frac{(1+i)^2}{2i+i^2} \cdot B$.

If $i = .10$, $d' = \frac{.21}{1.21}$, and the present value of the perpetuity is
$\frac{A}{d'} + \frac{1}{1+i} \cdot \frac{B}{d'} = \frac{1.21A}{.21} + \frac{1}{1.1} \cdot \frac{1.21B}{.21} = 5.761905A + 5.238095B = 24,619.05$.

If $i^{(2)} = .10$ then $i = .1025$, $d' = \frac{.215506}{1.215506}$, and the present value of the perpetuity is
$5.640235A + 5.115860B = 24,070.12$.

Solving these two equations for A and B results in $A = 2000$, $B = 2500$.

If $i^{(4)} = .10$ then $i = .103813$ and $d' = .179253$ and the present value is
$\frac{A}{d'} + \frac{1}{1+i} \cdot \frac{B}{d'} = \frac{2000}{.179253} + \frac{1}{1.103813} \cdot \frac{2500}{.179253} = 23,792$.